Electromagnetic tunneling through a single-negative slab paired with a double-positive bilayer

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We show that resonant tunneling of electromagnetic fields can occur through a three-layer structure composed of a single-negative (i.e., either negative permittivity or negative permeability) slab paired with a bilayer made of double-positive (i.e., positive permittivity and permeability) media. In particular, one of the two double-positive media can be chosen arbitrarily (even vacuum), while the other may exhibit extreme (either near-zero or very high) permittivity and permeability values. Our results on this counterintuitive tunneling phenomenon also demonstrate the possibility of synthesizing double-positive slabs that effectively exhibit single-negative-like wave-impedance properties within a moderately wide frequency range.

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The problem geometry is illustrated in the Cartesian (x,y,z) coordinate system of Fig. 1. Without loss of generality, we consider a homogeneous, isotropic ENG slab with relative permittivity \( \varepsilon_1 < 0 \) and thickness \( d_1 \) paired with a bilayer composed of homogeneous, isotropic DPS media with parameters \( \varepsilon_2 > 0 \), \( d_2 \) and \( \varepsilon_3 > 0 \), \( d_3 \), respectively, all embedded in vacuum (\( \varepsilon = 1 \)) and assumed as nonmagnetic (i.e., \( \mu = 1 \)) under time-harmonic \([\exp(-i\omega t)]\) plane-wave illumination.

Assuming normal incidence, for the ideal case of lossless media, we can derive via straightforward transfer-matrix algebra\(^{19}\) the general resonance condition for total transmission by zeroing the reflection coefficient, viz.,

\[
i(1 - \varepsilon_1)\sqrt{\varepsilon_2}\varepsilon_3 e_1 + i(1 - \varepsilon_2)\sqrt{-\varepsilon_1}\varepsilon_3 e_2 \\
+ i(1 - \varepsilon_3)\sqrt{-\varepsilon_1}\varepsilon_2 e_3 + \sqrt{\varepsilon_3}(\varepsilon_2 - \varepsilon_1)\tau_1 e_3 \\
+ \sqrt{\varepsilon_2}(\varepsilon_3 - \varepsilon_1)\tau_2 + \sqrt{\varepsilon_3}(\varepsilon_4 - \varepsilon_2)\tau_3 e_3 = 0,
\]

where \( \tau_1 = \tan(k\sqrt{-\varepsilon_1}d_1) \) and \( \tau_2, \tau_3 = \tan(k\sqrt{\varepsilon_3}d_2,3) \), with \( k = \omega/c = 2\pi/\lambda \) denoting the vacuum wave number, and \( c \) and \( \lambda \) denoting the corresponding wave speed and wavelength, respectively. Zeroing the real part of (1), we obtain

\[
\tau_2 = \frac{\sqrt{\varepsilon_3}(\varepsilon_2 - \varepsilon_1)}{\sqrt{\varepsilon_3}(\varepsilon_2 - \varepsilon_1)\tau_1 + \sqrt{\varepsilon_1}(\varepsilon_3 - \varepsilon_2)\tau_3},
\]

which, enforced in the imaginary part of (1), yields

\[
\tau_3 = \pm \frac{\varepsilon_3(\varepsilon_3 - \varepsilon_1)(\varepsilon_2 - \varepsilon_1)\tau_1^2}{\sqrt{\varepsilon_1}(1 - \varepsilon_3)(\varepsilon_3 - \varepsilon_2) + (\varepsilon_3 - \varepsilon_1)(\varepsilon_3\varepsilon_1 - \varepsilon_2)\tau_1^2},
\]

Noting that the numerator of the square-root argument in (3) is always positive (since \( \varepsilon_1 < 0 \) and \( \varepsilon_2,3 > 0 \)), real solutions exist if

\[
\varepsilon_1(1 - \varepsilon_3)(\varepsilon_3 - \varepsilon_2) + (\varepsilon_3 - \varepsilon_1)(\varepsilon_3\varepsilon_1 - \varepsilon_2)\tau_1^2 > 0.
\]

The above inequality is quadratic in \( \varepsilon_3 \) (with positive discriminant), and yields the conditions

\[
\varepsilon_3 \leq \varepsilon_{3a} \quad \text{or} \quad \varepsilon_3 \geq \varepsilon_{3b},
\]

with \( \varepsilon_{3a} \) and \( \varepsilon_{3b} \) denoting the two (positive) roots of the denominator of the square-root argument in (3). It can be
turns out to be end values \((3)\), it is readily verified that the solutions associated to the plus half-wavelength periodicities size for the transformation properties \(19\) is also representative of a different outermost DPS slab, which (recalling its wave-impedance tunneling configuration featuring an ENG slab paired with a tunneling configuration) are always a DPS half-space (i.e., \(\varepsilon = 3\)), the end values in \((5)\) tend to exhibit near-zero extreme values, so the outermost DPS slab must have either near-zero \((\varepsilon_{3a} \ll 1)\) or very-high \((\varepsilon_{3b} \gg 1)\) permittivity. From \((3)\), it is readily verified that the solutions associated to the end values \(\varepsilon_{3} = \varepsilon_{3a}\) or \(\varepsilon_{3} = \varepsilon_{3b}\) in \((5)\) correspond to a quarter-wavelength (plus half-wavelength periodicities) size for the outermost DPS slab, which (recalling its wave-impedance transformation properties\(19\)) is also representative of a different configuration featuring an ENG-DPS bilayer terminated with a DPS half-space (i.e., \(d_{1} \rightarrow \infty\)). Moreover, we highlight that the complete arbitrariness in the choice of \(\varepsilon_{2}\) represents an important degree of freedom in the proposed configuration. In particular, choosing \(\varepsilon_{2} = 1\), we obtain another interesting tunneling configuration featuring an ENG slab paired with a DPS slab via a separating vacuum layer. Finally, it can be shown that enforcing a purely real (i.e., \(\pm 1\)) transmission coefficient results in

\[
(\varepsilon_{1}^{2} - \varepsilon_{1}\varepsilon_{2} - \varepsilon_{1}\varepsilon_{3} + \varepsilon_{2}\varepsilon_{3})^{2}
= (1 - \varepsilon_{1})(\varepsilon_{2} - \varepsilon_{1})\left[\frac{\varepsilon_{1}}{\varepsilon_{1}^{2}}(\varepsilon_{3} - \varepsilon_{2})(\varepsilon_{3} - 1) + (\varepsilon_{1} - \varepsilon_{3})(\varepsilon_{1}\varepsilon_{3} - \varepsilon_{2})\right],
\]

e.g., a quadratic equation in \(\varepsilon_{3}\), the solutions of which, enforced in \((4)\), yield the constraint

\[
\varepsilon_{1}^{2}(1 - \varepsilon_{1})(\varepsilon_{1} - \varepsilon_{2})^{3}\tau_{1}^{2} > 0,
\]

i.e., which is clearly impossible to fulfill with \(\varepsilon_{1} < 0\) and \(\varepsilon_{2} > 0\), thereby implying that no tunneling with zero phase delay is possible in our proposed configuration.

As a first example, we consider an ideal lossless configuration featuring an ENG slab with \(\varepsilon_{1} = -3\) and \(d_{1} = 0.1\lambda_{0}\) (here and henceforth, the subscript 0 identifies resonant frequency and wavelength quantities). We arbitrarily select \(\varepsilon_{2} = 2.5\), and readily derive from \((2)-(5)\) the remaining parameters: \(\varepsilon_{3} = \varepsilon_{3b} = 16, d_{1} = 0.0624\lambda_{0}\) [choosing the positive determination for \(\tau_{3}\) in \((3)\)], and \(d_{2} = 0.236\lambda_{0}\). For a normally incident (along the positive \(x\) direction) plane-wave excitation, Fig. 2 illustrates the resonant electric and magnetic field (normalized) longitudinal distributions, from which can be observed the total-transmission effect, achieved via a growing evanescent wave in the ENG layer and a standing wave in the DPS bilayer. The electric field peaks toward the center of the middle DPS layer, and the magnetic field peaks at its boundaries, similar to a Fabry-Perot etalon, although we consider here the presence of an inherently opaque material. It is insightful to compare the above phenomenon with the tunneling occurring in an ENG-MNG pair. From the ENG-MNG matching conditions [cf. Eq. \((8)\) in Ref. \(3\)], we can straightforwardly derive the constitutive parameters \(\varepsilon_{e} = 1\) and \(\mu_{e} = -0.33\) of the required MNG slab of same thickness \((d_{2} + d_{3})\) as the above.

It can be shown that, for an increasing opacity of the ENG medium (\(|\varepsilon_{1}| \gg 1\)), the end values in \((5)\) tend to exhibit extreme values, so the outermost DPS slab must have either near-zero \((\varepsilon_{3a} \ll 1)\) or very-high \((\varepsilon_{3b} \gg 1)\) permittivity. From \((3)\), it is readily verified that the solutions associated to the end values \(\varepsilon_{3} = \varepsilon_{3a}\) or \(\varepsilon_{3} = \varepsilon_{3b}\) in \((5)\) correspond to a quarter-wavelength (plus half-wavelength periodicities) size for the outermost DPS slab, which (recalling its wave-impedance transformation properties\(19\)) is also representative of a different configuration featuring an ENG-DPS bilayer terminated with a DPS half-space (i.e., \(d_{1} \rightarrow \infty\)). Moreover, we highlight that the complete arbitrariness in the choice of \(\varepsilon_{2}\) represents an important degree of freedom in the proposed configuration. In particular, choosing \(\varepsilon_{2} = 1\), we obtain another interesting tunneling configuration featuring an ENG slab paired with a DPS slab via a separating vacuum layer. Finally, it can be shown that enforcing a purely real (i.e., \(\pm 1\)) transmission coefficient results in

\[
(\varepsilon_{1}^{2} - \varepsilon_{1}\varepsilon_{2} - \varepsilon_{1}\varepsilon_{3} + \varepsilon_{2}\varepsilon_{3})^{2}
= (1 - \varepsilon_{1})(\varepsilon_{2} - \varepsilon_{1})\left[\frac{\varepsilon_{1}}{\varepsilon_{1}^{2}}(\varepsilon_{3} - \varepsilon_{2})(\varepsilon_{3} - 1) + (\varepsilon_{1} - \varepsilon_{3})(\varepsilon_{1}\varepsilon_{3} - \varepsilon_{2})\right],
\]

e.g., a quadratic equation in \(\varepsilon_{3}\), the solutions of which, enforced in \((4)\), yield the constraint

\[
\varepsilon_{1}^{2}(1 - \varepsilon_{1})(\varepsilon_{1} - \varepsilon_{2})^{3}\tau_{1}^{2} > 0,
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which is clearly impossible to fulfill with \(\varepsilon_{1} < 0\) and \(\varepsilon_{2} > 0\), thereby implying that no tunneling with zero phase delay is possible in our proposed configuration.

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losses, of the above tunneling phenomenon. Figures 4 and 5 show the transmittance as a function of the frequency, compared with the response (red dashed line) of an effective MNG slab with $\varepsilon_r = 1, \mu_r = -0.33$.

DPS bilayer to “compensate” the given ENG slab. Figure 3 compares the reflection-coefficient responses of the isolated DPS bilayer and such MNG slab (free standing in vacuum) as a function of frequency. As expected, both responses result in large reflectivity (due to the opacity of an MNG standalone layer) and perfectly match at the resonance frequency. In particular, the designed DPS bilayer is capable of providing the capacitive input impedance required to resonate with the ENG slab, ensuring total transmission. The reflection coefficients in Fig. 3 agree reasonably well within a moderate bandwidth (maximum difference of ~0.5% in magnitude and ~π/12 in phase, over a 10% bandwidth), ensuring that the designed DPS bilayer may effectively replace an MNG layer for a variety of applications.

Next, we move on to assessing the frequency and angular dependence, as well as the sensitivity to polarization and losses, of the above tunneling phenomenon. Figures 4 and 5 show the transmittance as a function of frequency and incidence angle (from the x axis, for both P and S polarizations), respectively, for three representative parameter configurations. We start from the same configuration in Fig. 2, but now assuming a more realistic Drude-type dispersive, lossy model for the ENG medium,

$$\varepsilon_1(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma_1},$$

(9)

with the plasma angular frequency $\omega_p$ and the damping coefficient $\gamma_1$ chosen so as to ensure $\text{Re}[\varepsilon_1(\omega_0)] \approx -3$ (with a loss tangent $\sim 10^{-2}$), and a nondispersive slightly lossy (loss tangent $= 10^{-3}$) model for the DPS layers. From Fig. 4 (black solid curve), a rather broad resonance is observed in this case, with a peak transmittance of nearly 97%, and a slow decay for higher frequencies attributable to the increasing transparency (approaching the plasma frequency) of the ENG slab. Also, the angular response (see Fig. 5) turns out not to be very selective, especially for P polarization. As previously highlighted, tunneling effects may be obtained, in principle, for arbitrary choices of the ENG slab parameters and the permittivity of the middle DPS layer. For an increased opacity of the ENG medium, as anticipated, the constitutive parameters of the outermost DPS layer tend to exhibit extreme values. For instance, considering an ENG slab with $\varepsilon_1 = -100$ and $d_1 = 0.01\lambda_0$, and $\varepsilon_2 = 12$, we obtain [choosing the larger end value $\varepsilon_3$ in (5)] $\varepsilon_3 = 63.7$. In Ref. 16, such (positive or negative) high-permittivity media were successfully synthesized at

FIG. 3. (Color online) (a) Reflection coefficient magnitude and (b) phase of the isolated DPS bilayer in Fig. 2 (blue solid line) as a function of the frequency, compared with the response (red dashed line) of an effective MNG slab with $\varepsilon_r = 1, \mu_r = -0.33$.

FIG. 4. (Color online) Black solid line: Transmittance as a function of the frequency for the parameter configuration in Fig. 2, but using for the ENG slab the dispersive model in (9) with $\omega_p = 2\omega_0, \gamma_1 = 3.75 \times 10^{-4}\omega_0$ (i.e., $\text{Re}[\varepsilon_1(\omega_0)] \approx -3$), and a loss tangent of $10^{-3}$ for the DPS layers. Also shown are the responses obtained for an increased opacity of the ENG slab $\{\omega_p = 10.05\omega_0, \gamma_1 = 9.8 \times 10^{-4}\omega_0\}$ in (9) (i.e., $\text{Re}[\varepsilon_1(\omega_0)] \approx -100$), and $d_1 = 0.01\lambda_0$ for $\varepsilon_2 = 12$ (with loss tangent $= 10^{-3}$) and $d_2 = 0.119\lambda_0$ (red dashed line), and $\varepsilon_2 = 1$ and $d_2 = 0.472\lambda_0$ (blue dotted line), with the high-permittivity outermost DPS layer described by the model in (10), with $\varepsilon_3 = 4, \lambda_3 = 3.081\lambda_0$, and $\omega_3 = 1.091\omega_0, \gamma_3 = 9.26 \times 10^{-4}\omega_0$ (i.e., $\text{Re}[\varepsilon_3(\omega_0)] = 63.7$), $d_3 = 0.036\lambda_0$, and $\omega_3 = 1.13\lambda_0$, $\gamma_3 = 1.33 \times 10^{-4}\omega_0$ (i.e., $\text{Re}[\varepsilon_3(\omega_0)] = 47.8$), $d_3 = 0.0313\lambda_0$, respectively. The inset illustrates the influence of losses in the ENG material on the peak transmittance (as a function of the loss tangent at resonance).
microwave frequencies with resonant inclusions. Assuming for
the outermost DPS medium a Lorentz-type dispersive, lossy
model
\[ \varepsilon_3(\omega) = \varepsilon_{3\infty} - \frac{\Lambda_3^2}{\omega^2 - \omega_3^2 + 2i\gamma_3\omega}, \]  
(10)
with the parameters (given in Fig. 4 caption) tuned so as to
ensure the required real part at the given frequency, we observe
in Fig. 4 (red dashed curve) a narrower bandwidth (with
peak transmittance of nearly 85% due to the larger sensitivity
to losses, and a smaller transmission peak attributable to
dispersion effects), and from Fig. 5 a flatter angular response
(especially for \( P \) polarization), which is a direct consequence
of the increased permittivity values. The resonant field
distributions, not shown here for brevity, are qualitatively
similar to those in Fig. 2, with intensity enhancements of nearly
a factor of 60. Also shown in Figs. 4 and 5 (blue dotted curves)
are the responses obtained for the same ENG slab, but choosing
\( \varepsilon_3 = 1 \), i.e., an ENG-vacuum-DPS configuration. While the
frequency response and the field distributions (again, not
shown for brevity) resemble the previous example, the angular
response is now much more selective (for both polarizations)
and material losses. Our results, which can be extended (along
the lines of Refs. 18 and 20) to more general (e.g., optical,
quantum-mechanical) asymmetrical tunnel barriers, also allow
direct comparisons with ENG-MNG paired configurations,
and can be understood in terms of the equivalent wave
impedance properties exhibited by a DPS bilayer and a
“matched” (according to Ref. 3) SNG slab. This observation
may suggest more general application scenarios of such a DPS
bilayer, for which SNG-like responses may be emulated via
simpler dielectric slabs, which may somehow compensate,
in a simple geometry, opaque ENG metamaterial slabs for
a variety of applications in which complementary metamaterials
are paired together. Also of interest is the possible
adaptation for applications to super-resolution imaging
schemes.

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