Transformation-optics generalization of
tunnelling effects in bi-layers made of
paired pseudo-epsilon-negative/
underline{mu}-negative media

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Abstract

Transformation media designed by standard transformation-optics (TO) approaches, based on
real-valued coordinate mapping, cannot exhibit single-negative (SNG) character unless such
character is already possessed by the domain that is being transformed. In this paper, we show
that, for a given field polarization, pseudo-SNG transformation media can be obtained by
transforming a domain featuring double positive (or double-negative) character, via complex
analytic continuation of the coordinate transformation rules. Moreover, we apply this concept to
the TO-based interpretation of phenomena analogous to the tunnelling effects observable in
bi-layers made of complementary epsilon-negative (ENG) and mu-negative (MNG) media, and
explore their possible TO-inspired extensions and generalizations.

Keywords: transformation optics, single-negative media, tunnelling

(Some figures in this article are in colour only in the electronic version)

1. Introduction and background

Transformation optics (TO) (see e.g. [1–5]) is a powerful
and systematic approach to the synthesis of metamaterials
with broad field-manipulation capabilities. Its technological
viability is rapidly getting established, in view of the
formidable advances in the fabrication of artificial materials
and metamaterials with controllable anisotropy and spatial
inhomogeneity properties. The reader is referred to [6–14]
for a sparse sampling of recent representative applications,
ranging from the celebrated ‘invisibility cloaking’ [7], to
hyper/superlensing scenarios [8–11], and electromagnetic
(EM) analogues of relativistic effects [12] and celestial-
mechanical phenomena [13, 14].

Standard TO is based on the design of a suitable real-valued
coordinate transformation, which produces the desired
field behaviour in a curved-metric fictitious space. Thanks to
the formal invariance of Maxwell equations, this behaviour
may be translated to a flat-metric physical space filled up
with an anisotropic, spatially inhomogeneous ‘transformation
medium’ [1–5].

The constitutive parameters of such transformation media
are systematically derived from those of the fictitious space via
the Jacobian matrix of the transformation (see, e.g. equation (1)
below), and this imposes some restrictions on the material
properties that can be obtained by transforming a given
scenario into another. For instance, in the standard TO
approach, the reciprocity character of the domain that is
being transformed cannot be changed [15]. Moreover, the
signs of the constitutive-tensor eigenvalues can either be
all preserved or all flipped [15]. Thus, e.g. transforming
a domain featuring a double positive (DPS) character,
In section 3, we outline the main analytical derivations, starting from tunnelling conditions. In section 4, we provide special cases those in [20–22], and suggest other TO-inspired epsilon-negative (ENG) and mu-negative (MNG) slabs, and in connection with bi-layers of (homogeneous, isotropic) DNG) character, via complex analytic continuation of the coordinate transformation. In this paper, we further elaborate on this concept, and address its application to the TO interpretation of non-reciprocal and indefinite media.

For instance, in [15] an extension based on ‘triple spacetime transformations’ has been proposed, which allows geometrical interpretation of non-reciprocal and indefinite media.

In [19], as a possible extension of a general class of transformation slabs, we speculated that, for a given field polarization, SNG transformation media could be obtained by transforming a domain featuring DPS or DNG character, via complex analytic continuation of the coordinate transformation. In this paper, we further elaborate on this concept, and address its application to the TO interpretation of the tunnelling effects observed in [20] in connection with bi-layers of (homogeneous, isotropic) epsilon-negative (ENG) and mu-negative (MNG) slabs, and their further generalization to (inhomogeneous, anisotropic) complementary media [21, 22]. Our results reproduce as special cases those in [20–22], and suggest other TO-inspired broad tunnelling conditions.

Accordingly, the rest of the paper is laid out as follows. In section 2, we introduce the problem scenario and formulation. In section 3, we outline the main analytical derivations, starting from the field calculation, and proceeding with the study of total-transmission conditions. In section 4, we provide a physical interpretation of the results, and illustrate their relationship with those in [20, 21]. In section 5, we illustrate some novel representative results, not amenable to those in [20, 21]. Finally, in section 6, we provide some brief conclusions and hints to future research directions related to these findings.

2. Problem formulation

In [19], in connection with a general class of (DPS or DNG) transformation slabs, we have theoretically speculated that SNG transformation media could be generated by transforming a domain featuring DPS or DNG character, via a suitable analytical continuation of the coordinate transformation in the complex plane. From a physical viewpoint, one intuitively expects such coordinate transformation to exhibit an in-plane purely imaginary character, in order to map a propagating field solution in the fictitious ‘above cut-off’ space into an evanescent ‘below cut-off’ one in the transformed (SNG) domain. In this framework, while a single SNG transformation slab would be of limited interest (being inherently opaque), it may be of interest to verify that paired (ENG–MNG) transformation-media configurations may support interesting tunnelling phenomena under proper matching conditions, in analogy to what was shown in [20, 21]. Accordingly, as illustrated in figure 1(a), we consider a transformation bi-layer which occupies the region \( |x| < d \) in a physical space \((x, y, z)\), characterized by the relative permittivity and permeability tensors

\[
\varepsilon_{\alpha}(x, y) = \mu_{\alpha}(x, y) = \det[J_{\alpha}(x, y)] J_{\alpha}^{-1}(x, y), \quad (1)
\]

where \( \alpha = 1 \) for \(-d < x < 0\), \( \alpha = 2 \) for \(0 < x < d\), the superscript \( T \) denotes matrix transposition, and \( J_{\alpha}(x, y) \) are the Jacobian matrices of the two-dimensional (2D) coordinate transformations

\[
x' = i\alpha u_\alpha y_\alpha(x), \quad y' = i\alpha v_\alpha y_\alpha(x) + i\nu_\alpha x_\alpha, \quad z' = z,
\]

from a fictitious (vacuum) space \((x', y', z')\). In (2), and henceforth, \( \nu_\alpha \neq 0 \) are real scaling parameters, \( u_\alpha(x) \) and \( v_\alpha(x) \) are arbitrary continuous real functions, an overdot denotes differentiation with respect to the argument, and a time-harmonic \( \exp(-i\omega t) \) dependence is assumed for all field quantities. Moreover, we assume that the derivatives \( \dot{u}_\alpha(x) \) are continuous and positive within the bi-layer region \( |x| < d \) (so as to avoid singularities in (2)). We point out that the transformation in (2) is neither the only one, nor the most general in order to achieve the sought SNG character, and that it was chosen in view of its close resemblance to the (real) transformation considered in [19], so as to exploit (with some minor variations) our previous analytical derivations.

As shown in [19], the constitutive tensors in (1) are real and symmetric, and hence they can be diagonalized, thereby providing a more insightful interpretation. Accordingly, by indicating with \((\varepsilon_\alpha, \nu_\alpha, z)\) the principal reference systems (constituted by the orthogonal eigenvectors), and with \( \Lambda_{\varepsilon\alpha} \) and
Table 1. Possible characters of the SNG transformation media in (3), depending on the field polarization and on the scaling parameter sign.

<table>
<thead>
<tr>
<th>Polarization</th>
<th>Parameter sign</th>
<th>( a_\alpha )</th>
<th>( a_\alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE</td>
<td>( \bar{\varepsilon}<em>{x\alpha} &lt; 0, \bar{\mu}</em>{x\alpha} &gt; 0, \bar{\mu}<em>{y\alpha} &gt; 0 ) ( \bar{\varepsilon}</em>{x\alpha} &gt; 0, \bar{\mu}<em>{x\alpha} &lt; 0, \bar{\mu}</em>{y\alpha} &lt; 0 )</td>
<td>(ENG)</td>
<td>(MNG)</td>
</tr>
<tr>
<td>TM</td>
<td>( \bar{\varepsilon}<em>{x\alpha} &gt; 0, \bar{\varepsilon}</em>{y\alpha} &gt; 0, \bar{\mu}<em>{x\alpha} &lt; 0, \bar{\varepsilon}</em>{y\alpha} &lt; 0, \bar{\mu}_{y\alpha} &gt; 0 )</td>
<td>(MNG)</td>
<td>(ENG)</td>
</tr>
</tbody>
</table>

\( A_{\alpha \beta} \), the in-plane eigenvalues, the diagonalized forms can be written as (see [19] for details)

\[
\tilde{\varepsilon}_\alpha(x, y) = \mu_\alpha(x, y) = \begin{bmatrix} A_{\alpha \alpha}(x, y) & 0 & 0 \\ 0 & A_{\alpha \beta}(x, y) & 0 \\ 0 & 0 & -a_\alpha \end{bmatrix},
\]

where

\[
\text{sgn}[A_{\alpha \alpha}(x, y)] = \text{sgn}[A_{\alpha \beta}(x, y)] = \text{sgn}(a_\alpha).
\]

Therefore, depending on field polarization and on the sign of the scaling parameters \( a_\alpha \), the corresponding transformation media may effectively behave as either ENG or MNG, as compactly summarized in Table 1. However, since they exhibit by construction \( \tilde{\varepsilon}_\alpha = \mu_\alpha \), the terms ‘ENG’, ‘MNG’, and ‘SNG’ may appear not entirely appropriate, and throughout the paper we shall refer to them as ‘pseudo-ENG’ (P-ENG), ‘pseudo-MNG’ (P-MNG), and ‘pseudo-SNG’ (P-SNG).

In what follows, generalizing the studies in [20, 21], we analyse the transposition properties of bi-layers made of paired P-ENG/P-MNG transformation media.

3. Analytical derivations

3.1. Field calculation

As a first step, we calculate analytically the EM response for (unit-amplitude) plane-wave incidence, and, without losing generality, we focus on the TM-polarized case (\( z \)-directed magnetic field). In the fictitious (vacuum) space, no reflection takes place, and the total field coincides with the incident one, which can be written as

\[
H_z(x', y') = \exp[i(k_{00}x' + k_{y0}y')],
\]

where, for a propagating wave with incidence angle \( \theta_i \) with respect to the \( x \)-axis (see figure 1), the \( x \)- and \( y \)-domain wavenumbers are given by

\[
k_x = k_0 \cos \theta_i, \quad k_y = k_0 \sin \theta_i,
\]

with \( k_0 = 2\pi/\lambda_0 \) denoting the vacuum wavenumber (and \( \lambda_0 \) the corresponding wavelength).

In the standard TO approaches, the field in the physical space would be obtained by straightforward coordinate mapping (via (2)) of the fictitious-space expression in (5). However, in view of the discontinuity of the coordinate transformation in (2) at the interfaces \( x = 0, x = \pm d \), such mapping is not straightforward here, and special care is needed. Specifically, the physical-space field can be written as a superposition of forward- and backward-propagating coordinate-mapped (via (2)) plane waves, which can be compactly written as

\[
H_\pm(x, y) = A_\alpha^+ \exp[i(k_{x0}x' + ky_0y')] + A_\alpha^- \exp[-i(k_{x0}x' + ky_0y')],
\]

with \( A_\alpha^+ = 1, 2 \) labelling the bi-layer region (cf the coordinate mapping in (2)), and \( \alpha = 0 \) and \( \alpha = 3 \) labelling the surrounding vacuum regions \( x < -d \) and \( x > d \), respectively, where an identity transformation \( (x' = x, y' = y) \) is assumed. In (7), the amplitude coefficients \( A_\alpha^+ \), \( \alpha = 0, 3 \), as well as the wavenumbers \( k_x \) and \( k_y \), \( \alpha = 1, 3 \), need to be computed by enforcing the boundary conditions. First, we note that \( A_\alpha^+ = 1 \), consistently with the assumed unit-amplitude excitation (cf (5)), whereas \( A_\alpha^0 = 0 \) in order to fulfill the radiation condition. Next, by enforcing the phase-matching conditions at the interfaces \( x = \pm d \) and 0, we obtain

\[
k_{y1} = -ik_{y0}u_1(-d), \quad k_{y2} = -ik_{y0}u_1(d), \quad k_{y3} = k_{y0}u_1(0)u_2(d)/u_1(0),
\]

\[
k_{x\alpha} = \sqrt{k_0^2 - k_{y\alpha}^2}, \quad \text{Im}(k_{x\alpha}) \geq 0, \quad \alpha = 1, 3,
\]

which yield all the unknown wavenumbers in (7) as a function of the \( y \)-domain (vacuum) wavenumber \( k_{y0} \) in (6). Note that, consistently with our original intuition, the complex mapping in (2) transforms purely imaginary exponentials in the fictitious space, into real ones (with respect to the \( x \)-variable) in the bi-layer region \( (\alpha = 1, 2) \) of the transformed domain.

Finally, by enforcing the continuity of the tangential field components at \( x = \pm d \) and 0 (with the tangential electric field derived from (7) via the relevant Maxwell’s curl equation), we obtain a linear system of six equations, whose (straightforward, but cumbersome) solution yields the remaining unknown amplitude coefficients \( A_0^+, A_1^+, A_2^+, A_0^-, A_1^-, A_2^- \). Here, we focus on the coefficient \( A_0^+ \), which plays the role of the reflection coefficient, of direct interest for the subsequent developments, and can be written as

\[
A_0^+ = \frac{x_0 + ix_1 \tanh \kappa_{10} + ix_2 \tanh \kappa_{20} + x_{12} \tanh \kappa_{10} \tanh \kappa_{20}}{\xi_0 + ix_1 \tanh \kappa_{10} + ix_2 \tanh \kappa_{20} + \xi_{12} \tanh \kappa_{10} \tanh \kappa_{20}},
\]

\[
\times \exp(-2ik_{y0}d),
\]

where

\[
\kappa_{10} = a_{11}k_{x1}[u_1(0) - u_1(-d)],
\]

\[
\kappa_{20} = a_{22}k_{x2}[u_2(d) - u_2(0)],
\]

\[
x_0 = k_{x1}k_{x2} \left[ \frac{k_{x3}}{u_1(-d)u_2(0)} - \frac{k_{x0}}{u_1(0)u_2(d)} \right],
\]

\[
x_1 = k_{x2} \left[ \frac{k_{x0}k_{x3}}{u_2(0)} + \frac{k_{x2}^2}{u_1(-d)u_1(0)u_2(d)} \right],
\]

\[
x_2 = k_{x1} \left[ \frac{k_{x0}k_{x3}}{u_1(0)} + \frac{k_{x2}^2}{u_1(-d)u_2(0)u_2(d)} \right],
\]

\[
x_{12} = \frac{-k_{x2}^2k_{x3}}{u_1(0)u_2(-d)} - \frac{k_{x0}k_{x2}^2}{u_2(0)u_2(d)}.
\]
3.2. Total-transmission conditions

Based on the analytical solution above, we can now derive the conditions for total transmission, by zeroing the reflection coefficient in (10), namely,

\[ \chi_0 + i\chi_1 \tanh \kappa_{10} + i\chi_2 \tanh \kappa_{20} + \chi_{12} \tan \kappa_{10} \tan \kappa_{20} = 0. \]  

(14)

In general, the above equation needs to be solved numerically, and, for a given bi-layer configuration, its solutions (if any, and provided they are not roots of the denominator in (10) too) yield the conditions, in terms of the \( \kappa_i \) wavenumber values (of generally nonzero argument) in (14) cannot be exactly represented in terms of the functions \( \chi_i \) which turn out to be necessary and sufficient in order to obtain total transmission for any value of \( \kappa_{10} \), as it can also be verified that they do not imply the vanishing of the denominator in (10). Note that the conditions in (21) depend on the scaling parameters \( a_\alpha \) and on the boundary values of the functions \( u_\alpha \) and their derivatives, but are independent of the actual form of these functions as well as of the functions \( v_\alpha \). Moreover, while they may be interpreted as angle-independent conditions, they implicitly depend on frequency, in view of the unavoidable temporal-dispersion effects in passive SNG media.

Recalling the assumed positive character of the derivatives \( u_\alpha \), which implies that \( u_1(0) > u_1(-d) \) and \( u_2(d) > u_2(0) \), it readily follows from the third condition in (21) that

\[ \text{sgn}(a_1) = -\text{sgn}(a_2), \]  

(22)

and thus, depending on the field polarization (see table 1), we obtain two possible configurations corresponding to the P-ENG/P-MNG and P-MNG/P-ENG pairings.

Under the total-transmission condition, the total phase accumulated by the field transmitted through the bi-layer can be written as

\[ \Psi = -k_0u_1(-d)[v_1(0) + v_1(-d) + v_2(0) - v_2(d)], \]  

(23)

and therefore, at normal incidence \( (k_{10} = 0) \) for arbitrary functions \( v_\alpha \), or at arbitrary incidence for \( v_2(0) - v_2(d) = v_1(0) - v_1(-d) \), the field will undergo a complete tunnelling through the bi-layer, without any phase delay. The bi-layer thus behaves as an EM ‘nullity’, in close resemblance with the case of conjugate matched (homogeneous, isotropic) ENG–MNG pairs in [20], and its (inhomogeneous, anisotropic) complementary-media generalization in [21, 22] (see also the discussion in section 4). More in general, an arbitrary aperture field distribution located at a source-plane \( x = x_s < -d \) will ideally produce a perfect virtual image at the plane \( x = x_i = x_s + 2d \), apart from a possible rigid translation of \( y_0 = u_1(-d)[-v_1(0) + v_1(-d) + v_2(0) - v_2(d)] \) along the \( y \)-axis.

4. Physical interpretation

4.1. Special cases

In order to understand the physical mechanisms underlying the rather general class of transparent SNG transformation bi-layers identified by the conditions in (21), it is insightful to start by explaining their relation with the homogeneous, isotropic ENG–MNG bi-layers investigated in [20], and their
further (inhomogeneous, anisotropic) generalizations within the framework on complementary media [21, 22].

In [20], it was shown that complete tunnelling (with zero phase delay) could be achieved by pairing two slabs (of thickness $d_1$ and $d_2$) made of homogeneous, isotropic ENG and MNG materials. In particular, under the conjugate-matching condition:

$$\varepsilon_2 = -\varepsilon_1, \quad \mu_2 = -\mu_1, \quad d_2 = d_1, \quad (24)$$

such tunnelling effects would occur for any value of the incident wavenumber, as long as the above condition (24) remains satisfied. Such conditions were further generalized in [21, 22] to the case of anisotropic, inhomogeneous (possibly lossy) materials satisfying the complementary-media condition, which, in our chosen reference system, can be expressed as

$$\varepsilon_2 = \begin{bmatrix} -\varepsilon_{1xx} & \varepsilon_{1xy} & \varepsilon_{1xz} \\ \varepsilon_{1yx} & -\varepsilon_{1yy} & -\varepsilon_{1yz} \\ \varepsilon_{1zx} & -\varepsilon_{1zy} & -\varepsilon_{1zz} \end{bmatrix},$$

$$\mu_2 = \begin{bmatrix} -\mu_{1xx} & \mu_{1xy} & \mu_{1xz} \\ \mu_{1yx} & -\mu_{1yy} & -\mu_{1yz} \\ \mu_{1zx} & -\mu_{1zy} & -\mu_{1zz} \end{bmatrix}. \quad (25)$$

It can easily be verified that our general class in (21) reduces to the complementary-media case in (25) for

$$a_2 = -a_1, \quad u_2(x) = -u_1(-x), \quad v_2(x) = v_1(-x), \quad (26)$$

which, in turn, yields the homogeneous, isotropic case in (24) for $u_2(x) = x$ and $v_2(x) = 0$.

In this framework, we point out that some seeming restrictions in our class (e.g. the fact that $\varepsilon_{1x} = \mu_{1y}$, compare with (1)), are only due to our original choice of transforming a vacuum fictitious space, and can accordingly be overcome by considering a more general filling (not necessarily homogeneous and isotropic, and possibly DNG).

4.2. Complex-mapping-induced surface modes

While the complementary-media arguments above provide an interesting interpretation of some special cases, the conditions in (21) define a broad class of configurations that do not generally satisfy the symmetry conditions in (24) or (25).

A deeper physical insight can be gained by looking at the analytical structure of the field inside the bi-layer region. As anticipated, one intuitively expects the in-plane purely imaginary character of the coordinate transformation in (2) to map a propagating field (in the vacuum fictitious space, cf (5)) into an evanescent one in the P-SNG transformed domain. However, when pairing two conjugate SNG transformation slabs, an exponentially growing behaviour may also occur. Specifically, by particularizing the general expression in (7)
under the total-transmission conditions in (21), we obtain

\[ J_\alpha(x, y) = \begin{cases} 
\cosh[k_\alpha(x)] \pm \frac{k_\alpha u_\alpha(-d)}{k_1} \sinh[k_\alpha(x)] 
\end{cases} \]

\[ \times \exp \left[ -ik_\alpha d + ik_\alpha y \right] u_\alpha(-d), \quad \alpha = 1, 2, \]

(27)

where

\[ k_\alpha(x) = \pm \omega_\alpha k_\alpha [u_\alpha(x) - u_\alpha(\mp d)]. \]

(28)

We note that the field exhibits unit amplitude at the interfaces \( x = \pm d \) (as expected, in order to match the incident and totally transmitted fields), while its magnitude inside the bi-layer region is always greater than unity, exponentially growing towards the interface \( x = 0 \), where it reaches its peak. Thus, the complex mapping in (2), under the total-transmission conditions in (21), transforms a propagating (constant-intensity) plane-wavefield in the vacuum fictitious domain (see figure 3(a)) into a surface mode localized at the interface \( x = 0 \) separating the two opposite-signed P-SNG slabs in the physical space (see figure 3(b)).

In the complementary-media case \([20, 21]\), the presence of surface modes has been observed and associated with
tunnelling phenomena. Our complex-mapping arguments above provide an alternative TO-based interpretation, and extend the association between surface modes and tunnelling phenomena beyond the complementary-media scenario.

5. Representative numerical results

As a representative example, we consider a configuration featuring a homogeneous, isotropic MNG slab ($\varepsilon_{1\ell} = \varepsilon_{1\nu} = 1$, $\mu_{1\ell} = -1$), i.e.

$$a_1 = 1, \quad u_1(x) = x, \quad v_1(x) = 0,$$

paired with an inhomogeneous, anisotropic P-ENG transformation slab ($\varepsilon_{2\ell} < 0$, $\varepsilon_{2\nu} < 0$, $\mu_{2\ell} = 1$), obtained via

$$a_2 = -1, \quad u_2(x) = d \left[ \frac{x}{d} - 3 \left( \frac{x}{d} \right)^2 + 6 \left( \frac{x}{d} \right)^3 - 3 \left( \frac{x}{d} \right)^4 \right],$$

where the polynomial coefficients in $u_2$ have been chosen so as to satisfy the total-transmission conditions in (21), as well as to ensure that $u_2(x) > 0$ within the interval $0 < x < d$. Figure 3 illustrates the distributions of the relative-permittivity principal components of the P-ENG transformation slab, from which the inhomogeneous, anisotropic (and, obviously, negative) character is evident. We highlight that the bilayer configuration defined by (29) and (30) does not satisfy the symmetry conditions in (24) or (25), and thus its EM response cannot be explained within the complementary-media framework.

Figure 4 illustrates the EM response to a normally incident ($\theta_i = 0$), unit-amplitude plane-wave illumination, for a thickness $d = \lambda_0/3$. Specifically, figures 4(a) and (b) show the field magnitude (with stream lines illustrating the local energy flux) and phase maps, respectively, whereas figure 4(c) shows a longitudinal cut of the field magnitude. One can readily observe the predicted localization effects (surface mode) at the interface $x = 0$ (better quantified in the longitudinal cut), as well as the perfect matching between the phase profiles at the input ($x = -d$) and output ($x = d$) interfaces, indicative of the zero phase delay induced by the bi-layer.

Qualitatively similar considerations hold for the case of oblique incidence ($\theta_i = 15^\circ$), illustrated in figure 5, with an expected increased complexity in the internal field structure and energy flux.

We also investigated the effects of the unavoidable material losses in the above illustrated tunnelling phenomena (see [20, 22] for the complementary-media case). With reference to the parameter configurations in figures 4 and 5, figure 6 shows the transmission coefficient magnitude as a function of the material loss-tangent $\tan \delta$ (assumed identical for the two constituents of the bi-layer). The lossy configuration was obtained as a perturbation of the lossless one above, by multiplying all the negative constitutive parameters ($\mu_{1\ell}$, $\varepsilon_{2\ell}$, $\varepsilon_{2\nu}$) by a factor $(1 - i \tan \delta)$, so as to ensure the positivity of their imaginary parts, and hence the passivity of the media. As can be observed, the tunnelling effects turn out to be rather robust up to moderate (loss-tangent $= 0.01$) levels of losses. Being inherently a resonant phenomenon, however, significant metamaterial losses may significantly affect the tunnelling, as confirmed in figure 6. Analogous to the case of complementary SNG and DPS media, for which interesting tunnelling effects may occur at given frequencies and incidence angles.

6. Conclusions

In this paper, we have presented a TO-based interpretation of tunnelling phenomena in bi-layers made of paired P-ENG/P-MNG media. Our proposed interpretation, which relies on a complex coordinate transformation, includes the (homogeneous, isotropic) scenario in [20] and its (inhomogeneous, anisotropic) complementary-media extensions in [21, 22], and suggests other interesting generalizations, thereby providing further evidence of the versatility and unifying character of TO. Current and future research is aimed at the exploration of different TO-inspired scenarios, such as bi-layers made of P-SNG and DPS media, for which interesting tunnelling effects may occur at given frequencies and incidence angles.

References


