A Model-Based Parameter Estimation Approach for Numerical Analysis of Single-Mode Optical Fibers

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***Abstract*— An efficient numerical method is proposed and implemented for the analysis of propagation characteristics of single-mode optical fibers with arbitrary refractive index profile. The method follows the concept of the so-called *model-based parameter estimation*, and the Padé algorithm is used to construct a low-order rational approximant of the spectral domain Green's function, obtained by solving a hierarchy of *static* problems. The sought eigenfrequency and field distribution are then estimated by computing, respectively, the dominant pole and the related residual of the rational approximant. A number of profiles are analyzed and experiments show that very accurate results can be cheaply obtained through this technique.**

***Index Terms*— Model-based parameter estimation, optical fibers, Padé approximants.**

I. Introduction

NUMERICAL modeling of guided waves in *weakly guiding*, single-mode optical fibers plays an important role in the design of optical communication and computing systems. Although a number of index-profiles can be solved exactly, or using analytical approximations (e.g., Gaussian [1]–[3]), the need for ever enhanced performance motivates the investigation of novel profiles. Most design procedures rely on trial and error, to optimize the dispersion characteristics and match them to the spectral features of specific sources. The *bottleneck* of these procedures is related to the numerical solution of the scalar wave equation in the dominant-mode spectral range. Hence, developing fast and reliable alternative numerical techniques is a worthwhile issue, which has drawn considerable attention in the last decade, resulting in the proposal of several methods, including: piecewise-constant [4] or polynomial [5], [6] profile approximations; Gauss–Lauguerre modal expansions, possibly coupled to a Rayleigh–Ritz procedure for computing coefficients [7], [8]; integral equation (Galerkin [9] and finite element [10] methods); path-integral techniques [11], etc.

These methods suffer from various drawbacks. Matrix techniques [4], [9], [10] may be quite demanding in terms of memory space and computing times if highly accurate results are required. Polynomial expansions are often plagued by numerical instabilities [6]; Gauss–Laguerre expansions lead to nonlinear problems [7], [8]; path-integral (Monte Carlo) techniques [11], though being attractive in terms of easy implementation and little memory requirements, exhibit a slow convergency rate [11].

In this paper, we present a model-based parameter estimation (henceforth MBPE) approach [12], to obtain fairly accurate approximations for both the dispersion law and the field distribution of single-mode fibers, with a minimum storage and CPU budget.

MBPE is a powerful principle, widely applicable in electromagnetics, whereby mathematical tools *tailored* to the underlying physics are used to construct highly efficient representations/approximations of relevant problems' variables. As a typical example, we mention obtaining wideband responses from narrowband information [12]–[14].

In this spirit, we capitalize on the meromorphic property of the (azimuthally independent, spectral) Green's function, to construct from a straightforward Stevenson–McLaurin expansion, a (low-order) Padé (rational) approximant, whereby the dominant pole and related residual provide an accurate estimation of the dominant mode dispersion law and field distribution, respectively.

The remainder of the paper is organized as follows. In Section II the pertinent Sturm–Liouville problem is introduced, the (azimuthally independent) Green's function and its Stevenson–McLaurin expansion are derived, and used to construct a rational Padé approximant.

Numerical aspects and computational features are discussed in Section III. In Section IV, a number of examples are presented, comparisons with known (analytical) results are drawn to validate the method, and the relative merits and drawbacks of the proposed method, as compared to alternative techniques, are discussed. Concluding remarks follow under Section V.

II. Basic Principles

A. Azimuthally Independent Green's Function

Consider an arbitrary, longitudinally invariant, graded-index circular fiber whose refractive index profile has a uniform value $n_{\rm cl}$ in the cladding and an arbitrary radial variation in the core, as depicted in Fig. 1. The refractive index profile will be expressed in the general form [1]

$$n^2(r) = n_{\rm co}^2[1 - 2\Delta f(r)], \quad f(r) = \begin{cases} 0 & \text{for } r = 0 \\ 1 & \text{for } r \geq \rho \end{cases} \tag{1}$$

where r is the radial coordinate and $\Delta = \left(1 - n_{\rm cl}^2/n_{\rm co}^2\right)/2$ is the so-called *profile height parameter*, $n_{\rm co}$ being the maximum core index.

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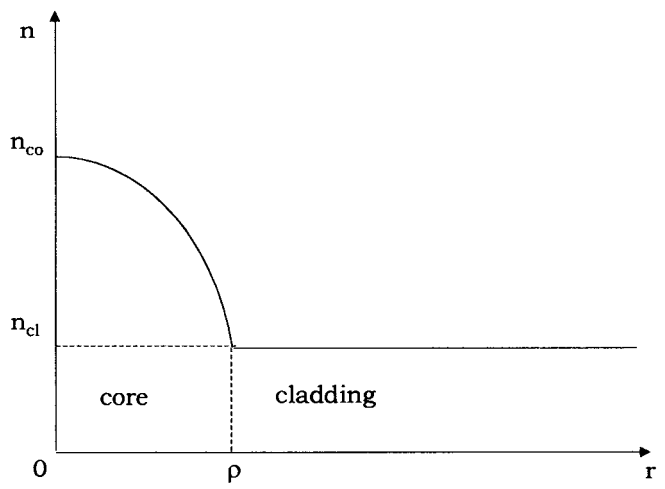

Fig. 1. Representative graded-index core profile $n(r)$—uniform cladding.

To cope with the existing notation [1] we use the following dimensionless parameters:

$$V = k\rho\left(n_{\rm co}^2 - n_{\rm cl}^2\right)^{1/2} \text{ normalized frequency,} \tag{2}$$

$$U = \rho\left(k^2 n_{\rm co}^2 - \beta^2\right)^{1/2}, \text{ core modal parameter} \tag{3}$$

$$W = \sqrt{V^2 - U^2}, \text{ cladding modal parameter} \tag{4}$$

wherein k is the free-space wavenumber, β is the propagation constant, and a time-harmonic dependence with angular frequency ω is assumed throughout.

Most fibers of practical interest in optical communication/computation systems are *weakly guiding* ($n_{\rm co} \approx n_{cl}$, i.e., $\Delta \ll 1$), and support nearly transverse electromagnetic (TEM) modes, whose spatial dependence is ruled by the *scalar* wave equation. For circular fibers, special attention is due to the azimuthally independent HE_{1n} modes, described by the following equation [1]:

$$\left[\frac{1}{R}\frac{d}{dR}\left(R\frac{d}{dR}\right) - W^2 + V^2 h(R)\right]\phi_n(R) = 0 \tag{5}$$

where $R = r/\rho$, $h(R) = 1 - f(R)$ and regularity-at-infinity boundary conditions are assumed. This problem belongs to the Sturm–Liouville class [15], and for any fixed positive (guided modes) value of the cladding parameter W, it admits a countable infinity of eigenvalues V_n^2 and eigenfunctions $\phi_n(\cdot)$ forming an orthonormal basis in $L^2(\mathcal{R}^+)$, viz. [15]

$$\int_0^1 R\phi_n(R)\phi_m(R)h(R)\,dR = \delta_{nm} \tag{6}$$

δ_{nm} being the Kronecker symbol.

The (frequency-domain) ring-source Green's function is ruled by [15]:

$$\left[\frac{1}{R}\frac{d}{dR}\left(R\frac{d}{dR}\right) - W^2 + V^2 h(R)\right] \cdot G_V(R, R') = \frac{1}{R}\delta(R - R'). \tag{7}$$

A straightforward Stevenson–McLaurin expansion of $G_V(\cdot)$ in powers of V^2, viz.

$$G_V(R, R') = \sum_{i=0}^{\infty} G_i(R, R')V^{2i} \tag{8}$$

is readily obtained. In fact, the Green's problem (7) admits the equivalent integral formulation [15]

$$G_V(R, R') = G_0(R, R') - V^2 \int_0^1 R'' G_0(R, R'') \times G_V(R'', R')h(R'')\,dR'' \tag{9}$$

where [15]

$$G_0(R, R') = \begin{cases} -I_0(WR)K_0(WR') & \text{for } R < R' \\ -K_0(WR)I_0(WR') & \text{for } R \geq R' \end{cases} \tag{10}$$

is the static solution of (7), and $I_0(\cdot)$, $K_0(\cdot)$ are the modified Bessel functions of zeroth order [16].

Using (8) in (9) one readily finds the recursive rule [15]

$$G_i(R, R') = -\int_0^1 R'' G_0(R, R'')G_{i-1}(R'', R')h(R'')\,dR'' \quad i \geq 1 \tag{11}$$

which can be alternatively rewritten as[1]

$$G_i(R, R') = (-1)^i \int_0^1 \int_0^1 \cdots \int_0^1 R_i h(R_i)G_0(R, R_1) \times G_0(R_i, R')\,dR_i \prod_{j=1}^{i-1} R_j G_0 \times (R_j, R_{j+1})h(R_j)dR_j, \qquad i \geq 1. \tag{12}$$

B. Model-Based Parameter Estimation: Padé Approximants

The azimuthally independent Green's function $G_V(\cdot)$ can be expressed exactly in terms of the eigenvalues V_n^2 and eigenmodes $\phi_n(\cdot)$, as follows [15]:

$$G_V(R, R') = \sum_{n=1}^{\infty} \frac{\phi_n(R)\phi_n(R')}{V^2 - V_n^2}. \tag{13}$$

The *meromorphic* structure of the Green's function (13) suggests that, in any *finite* spectral domain, it could be well approximated by a *rational* function of V^2, viz.

$$G_V(R, R') \approx \frac{\sum_{p=0}^{P} a_p(R, R')\,V^{2p}}{\sum_{q=0}^{Q} b_q(R, R')\,V^{2q}} =: G_V^{[P,Q]}(R, R'). \tag{14}$$

A systematic procedure to construct rational approximants starting from the *analytic element* (8), is obtained by exploiting the concept of *analytic continuation* and applying the Padé algorithm [17]. This technique provides a way for finding the

[1] An alternative procedure for computing the unknown $G_i(\cdot)$'s in (8) consists in using (8) into (7), and solving the resulting hierarchy of Sturm–Liouville problems by Frobenius method [15].

coefficients a_p, b_q in (14) such that the McLaurin expansion of (14) coincide up to order $(P+Q+1)$ with that of (8), i.e., [17]

$$\left(\sum_{q=0}^{Q} b_q(R, R')V^{2q}\right)\left(\sum_{i=0}^{\infty} G_i(R, R')V^{2i}\right) = \sum_{p=0}^{P} a_p(R, R')V^{2p} + \mathcal{O}\left(V^{2(P+Q+1)}\right) \quad (15)$$

where $\mathcal{O}$ is the Landau symbol. Equation (15) allows to determine the unknown coefficients a_p, $p = 0, 1, \cdots, P$ and b_q, $q = 1, 2, \cdots, Q$ unambiguously.[2]

In fact, by equating the coefficients of $V^0, V^2, V^4, \cdots, V^{2(P+Q)}$ on both sides of (15) the following equations are obtained:

$$\sum_{q=0}^{\min(Q,p)} b_q G_{p-q} = a_p, \qquad 0 \le p \le P \quad (16)$$

$$\sum_{q=0}^{\min(Q,p)} b_q G_{p-q} = 0, \qquad P \le p \le P+Q. \quad (17)$$

The rational approximant (14) whose coefficients are given by (16) and (17) is known as the (P, Q) Padé approximant (henceforth PA). A thorough description of the mathematical theory behind PA's, together with a discussion of their properties (existence, uniqueness, convergence) and applications may be found in [18] to [21]. We limit ourselves to quoting the following result [17]: if G_V is a rational function, with n (possibly coincident) zeros and m (possibly coincident) poles, then the Padé algorithm will reconstruct it *exactly*, $\forall\ P \ge n$ and $Q \ge m$.

Once a PA of (13) is computed, estimating the poles V_n^2 and the related residuals $\phi_n(R)\phi_n(R')$ is a rather trivial task, which can be accomplished by means of standard polynomial root-finding techniques.

In this paper we focus on *single-mode* fibers, and our goal will accordingly be computing the dispersion relation (usually represented as a V versus U diagram) and field distribution of the *fundamental mode* (HE_{11}). These latter, as seen from (13), can be obtained, respectively, from the first ($n = 1$, smallest module) pole and (but for a trivial scaling) related residual of $G_V(\cdot)$ in the complex V^2-plane.

The following questions arise naturally: i) How large should P, Q be in order to estimate the first pole V_1^2 and residual with sufficient accuracy? ii) To what extent can one eventually apply this technique to find out higher order poles and residuals (higher order eigenvalues and eigenfunctions of the fiber)?

So far, there appears to be no restriction on applying the algorithm to the analysis of higher order modes, by using approximants of higher order. Actually, this possibility is hindered by some numerical pitfalls. The linear system which yields the a_p's and b_q's is *ill-conditioned*, reflecting the fundamental *ill-posedness* of the analytical continuation problem.[3] Thus, unless the McLaurin coefficients in (8) are available in *exact, closed analytic form*,[4] unavoidable approximation and/or roundoff-errors in the Stevenson–McLaurin coefficients (12) set a limit to the spectral range where (14) is accurate.

On the other hand, numerical experiments confirm that as far as the estimation of the *dominant* pole (and related residual) is concerned, PA's of relatively low order are very robust[5] with respect to errors in the coefficients [24], [25]. In our double-precision numerical simulations we found, e.g., that (3, 3) PA's provided a reasonable tradeoff between accuracy and computing times.

III. Numerical Implementation

The main steps for implementing the proposed algorithm, to be repeated for a suitable number of values of W within the dominant-mode bandwidth, are as follows:

1) evaluate the McLaurin expansion coefficients using (12);
2) transform into a rational approximant via Padé technique;
3) extract dominant pole V_1^2 and residue $\phi_1(R)\phi_1(R')$ of the PA.

Accordingly, for each value of W a point of the dominant-mode dispersion curve $U = (V_1^2 - W^2)^{1/2}$ is obtained, and the (normalized) field distribution is deduced from $\phi_1(R)\phi_1(R')$ but for a trivial scaling.[6]

Step 1) is the most computationally demanding because it requires n-dimensional numerical quadratures $1 \le n \le (P + Q)$. In view of (10), it is convenient to split each one-dimensional (1-D) integral in (12), in two subintegrals where the integrand function is *smooth*. Then, e.g., a Gaussian quadrature formula [21] can be used to compute each subintegral. In our numerical experiments, we used three-point Gaussian quadrature formulas, which gave fairly accurate results. For the more general case of n_S-segmented profile fibers, the computational cost of (12) can be readily estimated. Each integral in (12) splits into $n_S + 1$ subintervals where the integrand function is smooth. If an n_G-point Gaussian quadrature formula is used for each subintegral, it is readily shown that the computational cost for evaluating the needed $P + Q$ coefficients goes (asymptotically) like $[n_G(n_S + 1)]^{P+Q} T_{G_0}$ where T_{G_0} is the CPU time required for evaluating G_0. If a recursive implementation is used, the corresponding memory budget is $n_G(P+Q)(n_s+1)$ double precision variables, which grows only linearly with the number of segments.

The remaining steps 2) and 3) have a negligible computational cost, and can be even performed analytically.

IV. Application Examples and Discussion

A computer code has been developed which implements the described algorithm. In order to validate the method we

[2] Note that b_0 can be always set equal to one.

[3] Ill-conditioning becomes more serious the higher the order of the approximant [21].

[4] This is the case, e.g., for dielectric slab waveguides with polynomial profiles.

[5] For a large class of functions and noise models, PA's coefficients may be even less noisy than the McLaurin coefficients used to compute them [22], [23].

[6] The obtained results are obviously independent on the choice of R'. A computationally convenient choice, in view of the possibility of using handy approximations of the Bessel functions in (10) is $0 \ne WR' \ll 1$.

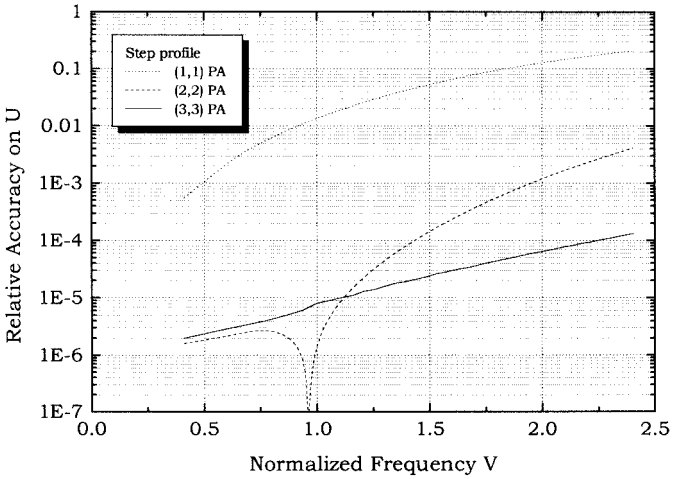

Fig. 2. Step profile—relative accuracy of approximate dispersion law (diagonal PA) over the dominant-mode frequency range.

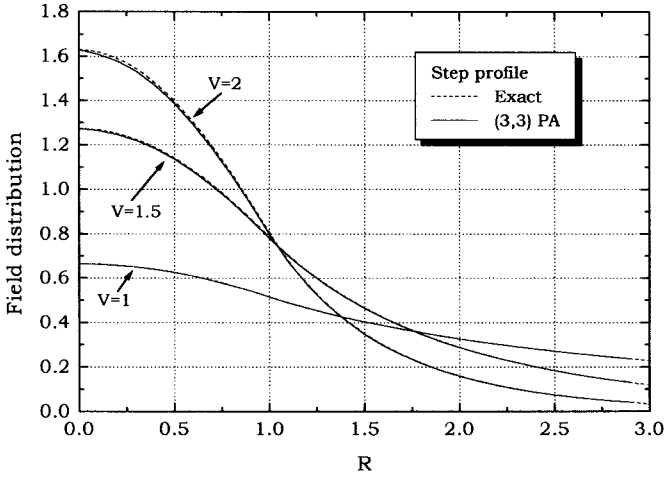

Fig. 4. Step profile—exact and (3, 3) PA field distributions at various frequencies.

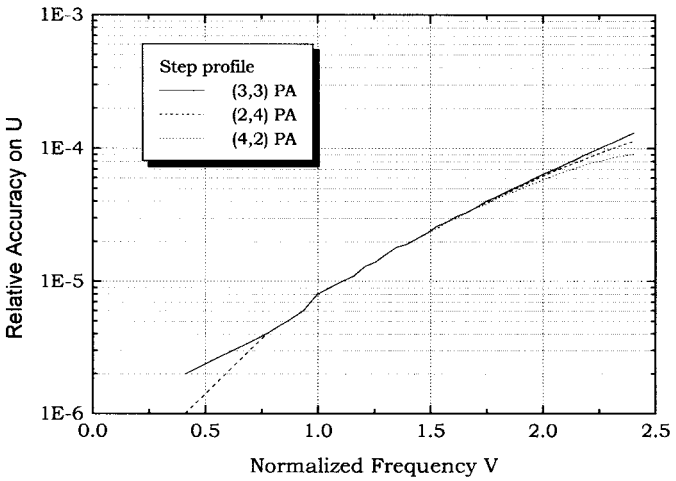

Fig. 3. Step profile—relative accuracy of approximate dispersion law (diagonal and nondiagonal PA's) over the dominant-mode frequency range.

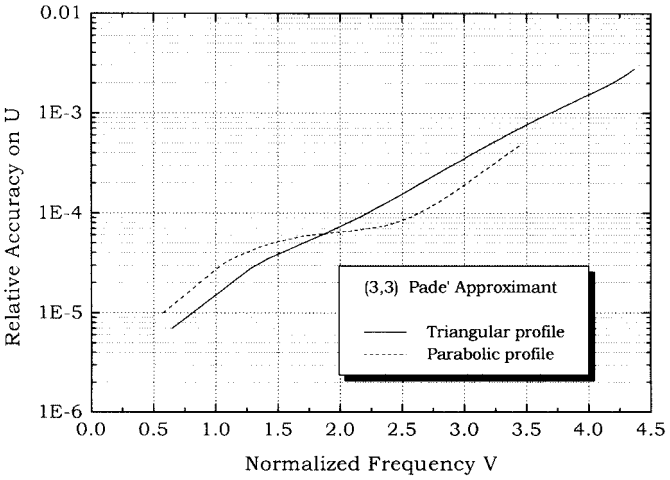

Fig. 5. Relative accuracy of approximate dispersion laws [(3, 3) PA] for triangular and parabolic profiles over the corresponding dominant-mode frequency ranges.

analyzed some simple refractive index profiles for which analytical solutions are known.

As a first example, we consider the step profile [1]. In Fig. 2 the relative accuracy of the dominant-mode dispersion law, obtained using various diagonal PA's, is reported. As one can see, the (3, 3) PA provides a (uniformly) high accuracy over the whole dominant mode bandwidth. The required computing time was pretty short (five seconds per point on a Pentium P200), providing a good tradeoff between accuracy and speed. It is interesting to note that, as shown in Fig. 3, given the order of the McLaurin expansion, diagonal and *nondiagonal* PA's, namely (3, 3), (2, 4), and (4, 2), provide comparable levels of accuracy. In the following we use diagonal approximants, since most of the known convergency theorems and conjectures apply to them [17]. In this connection, note also that

$$\lim_{V\to\infty} G_V^{[P,P]}(R, R') = \text{cost.} \neq 0. \tag{18}$$

At a first glance, this kind could appear *unphysical*. However, it has been recognized that the addition of an entire-function term (a constant, in our case) in reduced-order models, often can mimic the contribution of the high-frequency tails of the spectrum [12].

Fig. 4 displays the exact and approximate modal field distributions for the step profile at various frequencies. The agreement with the exact solution is also excellent.

As further examples, we consider two relevant power-law profiles, namely, the *triangular* profile, $f(R) = R, 0 < R \leq 1$ and the *parabolic* one, $f(R) = R^2, 0 < R \leq 1$. For both profiles we use as reference solution the power-series expansion (100 terms) reported in [1]. Results are displayed in Fig. 5 (accuracy on dispersion laws) and Figs. 6 and 7 (field distributions). Again, a very good accuracy is observed.

We conclude this section drawing a comparison between the proposed MBPE approach and the more usual ones in terms of computational aspects. The main merits of the proposed approach can be summarized as follows.

- Straightforward implementation (irrespective of the complexity in the refractive index distribution)—neither prior-

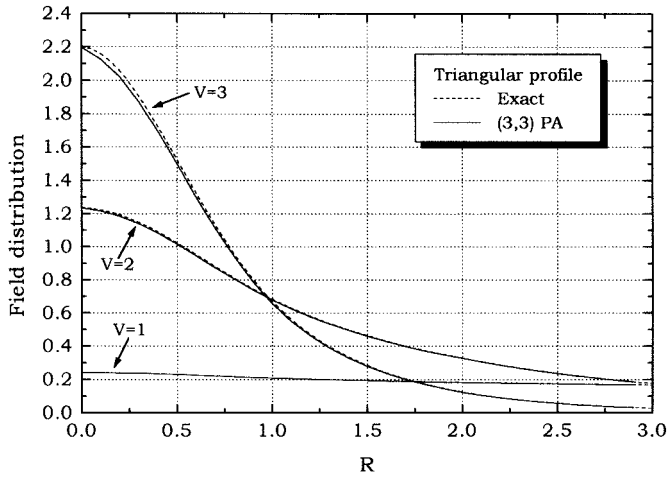

Fig. 6. Triangular profile—exact and (3, 3) PA field distributions at various frequencies.

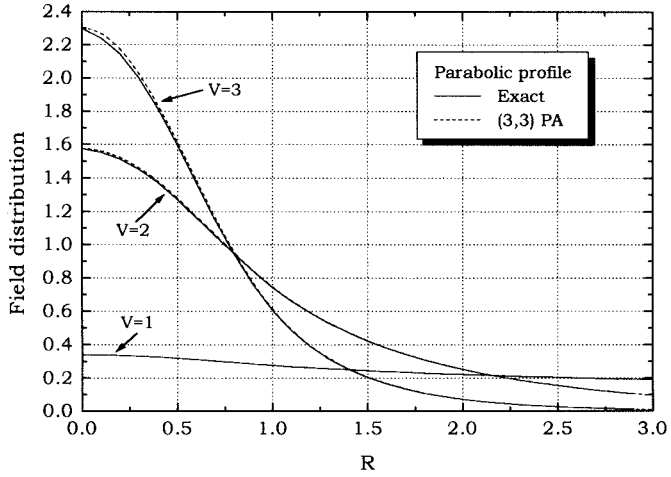

Fig. 7. Parabolic profile—exact and (3, 3) PA field distributions at various frequencies.

meshing, nor choice of basis (test) functions are required, in contrast to standard finite elements and Galerkin methods.

- Considerable saving in terms of memory and CPU budget as compared to matrix-based approaches—as an example, piecewise constant approximation of the profile typically requires 10^3–10^4 matching points to produce accurate results [4]. On the other hand, path-integral methods [11], though being comparable to the proposed approach in terms of implementation and memory requirements, turn out to be worse in terms of CPU times, at the same level of accuracy.
- Stability of the solution, as compared to methods based on polynomial (Chebyshev) approximations of the profile function [5].
- Complete linearization of the problem,[7] as compared to several implementations of Gauss–Laguerre expansion based algorithms [7], [8].

[7] In view of the relatively low order of the involved PA's, the required root-finding can be performed analytically.

Furthermore, it should be pointed out that, as expected (the approach being based upon a Stevenson–McLaurin expansion), the proposed method provides fairly accurate results in the low-frequency limit, where (standard or improved) Gaussian approximations are known to fail [1]–[3].

The main drawback seems to be related to the restriction to scalar problems and dominant modes only, although, as already stated, many problems of the practical interest cope with these limitations.

V. Conclusions and Recommendations

We presented an MBPE framework for analyzing arbitrary profile single-mode fibers, based upon Padé approximation of the spectral domain Green's function. Computational features and comparison with known analytical results confirm that the proposed method allows to achieve high accuracy over the single-mode spectral range with very mild memory and CPU requirements. The method seems to be very well suited for fast and reliable trial-and-error optimization of the refractive index profile.

Extension to several more general problems is possible, in principle, at the expense of formal (e.g., *vector* problems) and computational (higher order modes) complications. As a possible hint we mention using two-point PA's, obtained from both a McLaurin and an *asymptotic* (e.g., Lunenburg–Kline [26]) expansion of the Green's function, to obtain more uniform approximations over wider spectral ranges, including higher order modes [27]. Matrix regularization procedures, e.g., singular value decomposition or generalized inverses [21], could be used to control ill-conditioning.

As a last possible application, the method could be extended to compute the characteristic modes of dielectric antennas.

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