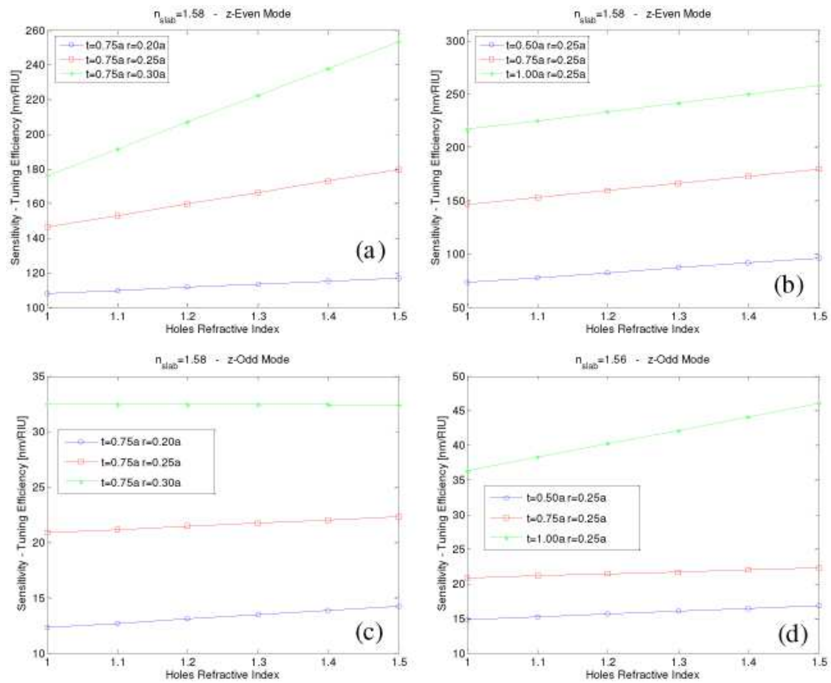

Fig. 8. As in Fig. 6, but for a low-index ($n_{\mathrm{slab}} = 1.58$) slab.

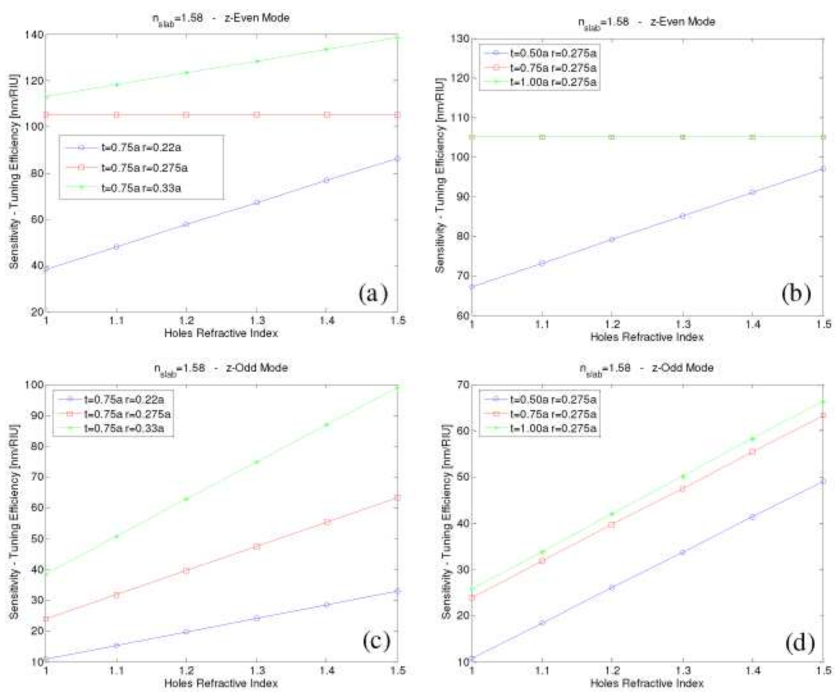

Fig. 9. As in Fig. 7, but for a low-index ($n_{\mathrm{slab}} = 1.58$) slab.

3.3. Perturbative model for GRs in PCs and PQCs

In view of the above illustrated complex dependence of the GR sensitivity on the slab refractive index, mode type (*z*-even or *z*-odd), and lattice geometry (PC or PQC), in what

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follows, we try to gain further insight in the relationship between the wavelength shift and the modal field distribution of the unperturbed (n_{holes} = 1) structures. In particular, we apply the perturbation theory, originally developed in connection with microwave cavity tuning [27], to model the dependence of GRs wavelength shift on the hole refractive index. In this framework, generalizing the results in [27, Sec. 7.3] to the case of a *fully dielectric* cavity composed of the unperturbed (n_{holes} = 1) PC or PQC slab surrounded by the air layers along the z-axis and terminated by periodic boundary conditions in all directions, the GR wavelength shift due to a variation in the hole refractive index (i.e., $n_{\text{holes}}\neq 1$) can be *rigorously* related to the unperturbed $(\boldsymbol{E}_0, \boldsymbol{H}_0)$ and perturbed $(\boldsymbol{E}, \boldsymbol{H})$ electric and magnetic modal field distributions as follows:

$$\Delta\lambda = \lambda_0 \frac{\left(n_{holes}^2 - 1\right)\iiint\limits_{holes} \boldsymbol{E}(\boldsymbol{r})\cdot\boldsymbol{E}_0^*(\boldsymbol{r})\,d\boldsymbol{r}}{\iiint\limits_{cavity}\left[n^2(\boldsymbol{r})\boldsymbol{E}(\boldsymbol{r})\cdot\boldsymbol{E}_0^*(\boldsymbol{r}) + \eta_0^2\boldsymbol{H}(\boldsymbol{r})\cdot\boldsymbol{H}_0^*(\boldsymbol{r})\right]d\boldsymbol{r}}, \tag{3}$$

where λ_0 and η_0 indicate the unperturbed GR wavelength and the vacuum characteristic impedance, respectively, * denotes complex conjugation, and

$$n(\boldsymbol{r}) = \begin{cases} n_{\text{slab}}, \text{in the slab,} \\ 1, \text{in the holes and background.} \end{cases} \tag{4}$$

Note that, albeit *exact*, Eq. (3) requires the complete knowledge of the *perturbed* modal field distribution, and so its practical applicability is very limited. However, in the limit of *small perturbations* ($n_{\text{holes}}\approx 1$), where one can assume $\boldsymbol{E} \approx \boldsymbol{E_0}, \boldsymbol{H} \approx \boldsymbol{H_0}$, it can be simplified to a form that involves only the electric modal field distribution of the *unperturbed* structure, viz.,

$$\Delta\lambda \approx \lambda_0 \frac{\left(n_{holes}^2 - 1\right)\iiint\limits_{holes}\left|\boldsymbol{E}_0(\boldsymbol{r})\right|^2 d\boldsymbol{r}}{2\iiint\limits_{cavity} n^2(\boldsymbol{r})\left|\boldsymbol{E}_0(\boldsymbol{r})\right|^2 d\boldsymbol{r}}, \tag{5}$$

where use has been made of the equality between the resonant unperturbed magnetic and electric energy densities.

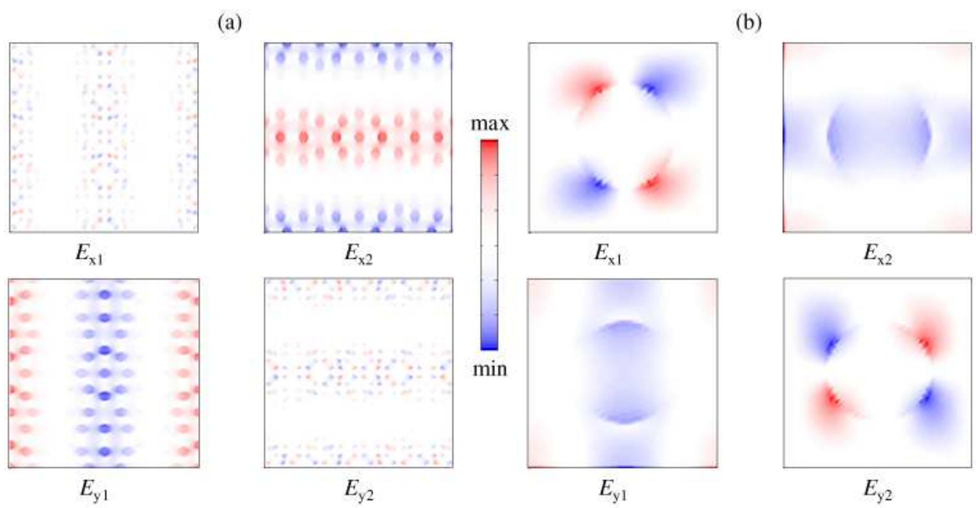

Fig. 10. Electric field amplitudes (at the slab center x-y plane) for x- and y-components of the lowest order z-even degenerate modes, in the PQC (a) and PC (b) case, with high-index slab.

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From the physical viewpoint, Eq. (5) states that the GR wavelength response to variations of the holes refractive index strongly depends on the electric field concentration in the slab holes. In order to assess to what extent such model is able to capture and explain the observed differences among GRs associated to z-even and z-odd modes, in PCs and PQCs with high- and low-index slabs, we compare the wavelength shift calculated according to Eq. (5) with the corresponding results from our previous *full-wave* study. For brevity, we restrict this comparison to the configuration that was observed to provide the highest sensitivity, i.e., the lowest-order z-even mode in the PQC supercell with high-index slab ($n_{slab} = 3.418$). In particular, we refer to the configuration with hole radius $r = 0.25a$ and slab thickness $t = 0.75a$, whose modal field distributions have already been extensively investigated in [8]. In Fig. 10(a) we recall the in-plane electric-field x- and y- components of the two degenerate modes (E_{xj} and E_{yj}, with $j = 1,2$), exhibiting the mirror symmetries that allow the coupling with a normally-incident (y- or x-polarized, respectively) plane wave. Similarly, in Fig. 10(b), we show the PC counterparts, pertaining to a structure with hole radius $r = 0.275a$ [cf. Equation (1)].

In Fig. 11, the GR wavelength shifts resulting from our previous full-wave modal analysis are compared with those predicted from the numerical implementation of the perturbative model in Eq. (5). A substantial agreement in the trends can be observed, thereby confirming that the previously highlighted differences between PCs and PQCs in terms of sensitivity-tuning efficiency are actually captured by the simple perturbative model in Eq. (5).

To sum up, by applying simple perturbation arguments to GRs in PC and PQC slabs, we ascertained that the GR sensitivity to variations in the hole refractive index is basically dictated by the electric field concentration in the holes. Therefore, the counterintuitive differences emerged in our parametric studies are basically attributable to different field distributions in the slab holes.

The above analysis, besides offering a physical insight in the observed differences, also suggests useful prediction/design strategies which rely on the field distribution of the unperturbed structure. In this framework, the numerical results suggest also that the spatial arrangement of the holes (and, in particular, the *aperiodic* arrangement in PQC-type supercells) represents an important degree of freedom to increase the interaction with the material infiltrated in the holes.

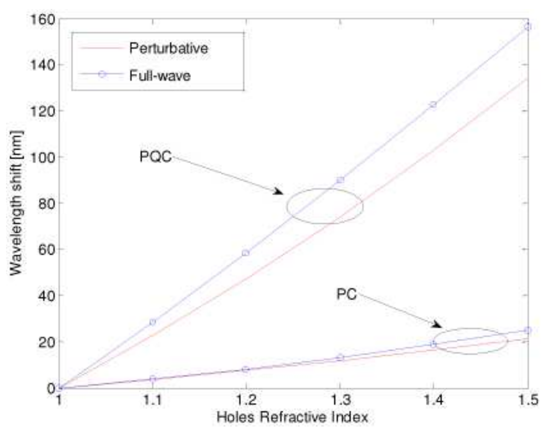

Fig. 11. GR wavelength shift calculated via full-wave modal analysis and perturbative model [cf. Equation (5)], pertaining to the lowest order z-even mode, in the PC ($r = 0.275a$) and PQC ($r = 0.25a$) case, with a high-index ($n_{slab} = 3.418$) slab of thickness $t = 0.75a$.

4. Conclusions

In this paper, we have presented a comparative numerical study of the GR properties in PC and PQC slabs constituted by arrangements of holes (with periodic and aperiodically-ordered supercells, respectively) in a dielectric host medium. The analysis has been focused on the GR dependence on the hole refractive index, and comparisons have been drawn in terms of GR wavelength sensitivity and tuning efficiency. Such sensing/tuning scheme is representative of a device in which a sensitive/active material is infiltrated in the slab holes.

Our numerical results highlighted the dependence of the GR sensitivity-tuning efficiency on the physical and geometrical parameters characterizing the patterned slab, as well as on the nature of the GR modes (z-even or z-odd). Specifically, for the parameter settings considered, the best performance was achieved with GRs associated to the lowest-order z-even mode in a PQC slab with n_{slab} = 3.418. It is important to remark that, although in the specific context investigated in this work, PQC-based GRs turn out to exhibit higher sensitivity than their periodic counterpart, a general rule cannot be derived at this stage, since a deeper analysis is necessary, involving different tiling geometries (e.g., Penrose, dodecagonal, etc.) for the PQC case and different symmetries (e.g., hexagonal) for the periodic case.

Finally, by applying simple perturbation arguments, we have investigated the dependence of the GR wavelength shift on the modal field distribution of the unperturbed (air holes) structure. Overall, our results indicate that the GR sensitivity to variations in the hole refractive index is basically dictated by the electric field concentration in the holes, and that, within the investigated context, the hole arrangement in the PQC case provides a significant improvement of the light-matter interaction via a stronger field localization in the holes. Moreover, the agreement between the numerical results and those obtained through the perturbative approach suggests a systematic approach for the design and optimization of highly sensitive/active devices and components by acting on the hole spatial arrangement in the unperturbed structures.

Current and future studies are aimed at the exploration of other PC and PQC geometries (based on aperiodic tilings or substitutional sequences), as well as at the experimental verification of the results and their application to the design of high-performance sensing/active devices.

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