Transformation Media for Thin Planar Retrodirective Reflectors

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***Abstract*—In this letter, we address the design of thin planar retrodirective reflectors via transformation optics. Exploiting form-invariant transformations of Maxwell's equations, we derive the constitutive properties of an anisotropic and spatially inhomogeneous "transformation-medium" coating which, laid on a flat metallic surface, exhibits the retrodirective response typical of a dihedral corner reflector. The practical feasibility of the involved materials should be within reach of the fast-pacing "metamaterial" technology. The results of our investigations, validated via a full-wave study of the near- and far-field responses, could find interesting applications in radar, communication, and identification scenarios.**

***Index Terms*—Transformation media, metamaterials, corner reflectors, scattering.**

I. Introduction and Background

THE rapid advances in the engineering of "metamaterials" with controllable anisotropy and spatial inhomogeneity have rendered "transformation optics" [1], [2] a viable and promising technique for controlling the electromagnetic (EM) response of devices and components. Previously inconceivable applications such as the EM "invisibility cloaking" have been devised and experimentally verified at microwave frequencies [3], and other exciting developments are foreseen in a wide range of applications including EM "wormholes" [4], rotators [5], "masking" [6], waveguide transitions and bends [7], low-profile radiators [8], and lenses [9].

In this letter, we apply transformation optics to the design of thin planar reflectors with *retrodirective* response (i.e., capable of reflecting most of the impinging energy back toward the source). As is well known, such a response, desirable in many radar, communication and identification applications, is not naturally exhibited by standard planar metallic surfaces, which instead tend to reflect *specularly*. Following the transformation-optics approach, we first design the desired field behavior in a fictitious space with a suitable (nonflat) topology. Next, by exploiting the formal invariance of Maxwell's equations under coordinate transformations, we interpret such a behavior in a conventional flat, Cartesian space filled by a suitable (anisotropic and spatially inhomogeneous) "transformation medium."

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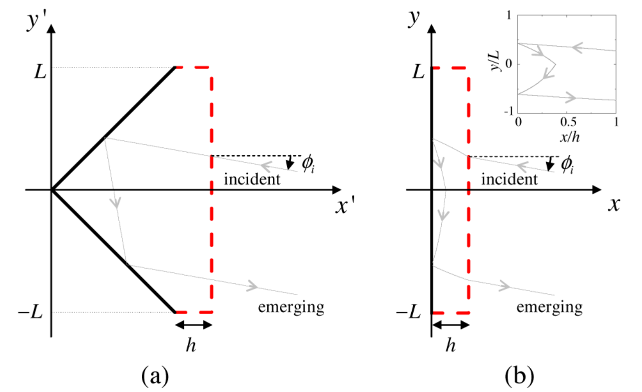

Fig. 1. (Color online) Problem geometry. (a) Dihedral corner reflector in the (x', y') space. (b) Proposed flat reflector in the transformed (x, y) space. Black thick lines denote the PEC boundaries; red dashed lines delimit the free-space (a) and coating (b) regions involved in the transformation; grey thin arrowed lines indicate typical ray trajectories. The zoom in the inset highlights the negative refraction occurring in the transformation medium at the $y = 0$ interface.

II. Geometry, Observables, and Results

Our coordinate transformation is inspired by the possibly simplest conceivable geometrical shapes that naturally exhibit a retrodirective response: the *corner reflectors* [10]. Fig. 1(a) illustrates the so-called *dihedral* configuration, of interest for our two-dimensional (2-D) prototype study, composed of two orthogonal infinitely-long perfectly-electric-conducting (PEC) sheets (black thick lines) of width $\sqrt{2}L$ (i.e., with a total aperture of $2L$) in free space. Its retrodirective response is readily understood in the ray-optical limit: as exemplified in Fig. 1(a) (grey thin arrowed lines), for incidence angles $|\phi_i| < 45^\circ$ and a sufficiently large corner, the trajectory of a doubly-reflected ray is *always* parallel to the incidence direction. A possible coordinate transformation capable of "flattening" the above geometry is given by

$$x = \frac{h\,(x' - |y'|)}{L + h - |y'|}, y = y', z = z', |y'| \le L, \tag{1}$$

with the parameter h controlling the aspect-ratio of the transformed geometry. Fig. 1(b) illustrates the mapping, via (1), of the corner reflector plus a portion of free-space of thickness h [delimited by the red dashed lines in Fig. 1(a)] in the original (x', y', z') space onto a *flat* ground plane of width $2L$ (black thick line) topped by a rectangular slab of thickness h (red dashed lines) in the transformed (x, y, z) space; also shown (grey thin arrowed lines) is the transformed ray trajectory, which exhibits "negative refraction" at the $y = 0$ interface (evidenced by the zoom in the inset), and preserves the retrodirective behavior.

Following [1] and [2], in order for the flat configuration in Fig. 1(b) to exhibit the same EM response as the original corner reflector in free space [Fig. 1(a)], the transformed region needs to be filled up with a transformation medium whose relative permittivity and permeability tensors are given by

$$\underline{\underline{\varepsilon_r}} = \underline{\underline{\mu_r}} = \underline{\underline{J}} \cdot \underline{\underline{J}}^T \left[\det\left(\underline{\underline{J}}\right)\right]^{-1} \tag{2}$$

where $\underline{\underline{J}} = \partial(x,y,z)/\partial(x',y',z')$ denotes the Jacobian matrix of the transformation in (1), and the superfix^T denotes transposition. After straightforward algebra, we obtain the explicit expression

$$\underline{\underline{\varepsilon_r}}(x,y) = \underline{\underline{\mu_r}}(x,y) = \begin{bmatrix} \frac{h\left[1+\left(\frac{x}{h}-1\right)^2\right]}{L+h-|y|} & \operatorname{sgn}(y)\left(\frac{x}{h}-1\right) & 0 \\ \operatorname{sgn}(y)\left(\frac{x}{h}-1\right) & 1+\left(\frac{L-|y|}{h}\right) & 0 \\ 0 & 0 & 1+\left(\frac{L-|y|}{h}\right) \end{bmatrix} \tag{3}$$

where $\operatorname{sgn}(\cdot)$ denotes the signum function, and the anticipated anisotropy and spatial inhomogeneity are evident. Note that the perfect-impedance matching (i.e., zero reflection) at the interface with free-space is guaranteed by the structure of the coordinate-transformation in (1), which is chosen so as to act on that interface as a rigid translation. For a more incisive physical interpretation, and in view of its symmetry, it is expedient to diagonalize the tensor in (3). Fig. 2 shows, for a coating thickness $h = L/(3\sqrt{2})$, the spatial distribution of the diagonalized tensor components [eigenvalues of (3)] in the local principal coordinate system (u,v,z), with u and v (shown as short segments) denoting the eigenvectors of (3). We observe that the material parameter distributions and the principal axes u,v exhibit reflection symmetry around the x-axis, inherited by the original corner-reflector geometry. In this connection, the negative refraction observed for the ray trajectory inside the coating at the $y = 0$ interface [Fig. 1(b)] could be interpreted similarly as the peculiar refraction effects observed in "twinning" structures [11]. We also note that the values of the material parameters are *everywhere finite* and not particularly high. The actual feasibility of such a medium, possibly via simplified parameters [3], [12] should therefore be within reach of the current metamaterial technology.

We studied the near- and far-field responses of this configuration, for time-harmonic $(\exp(j\omega t))$ plane-wave excitation with unit-amplitude $z-$directed electric field. For computing the near-field response, we used a full-wave finite-element-method (FEM) commercial software [13] capable of handling anisotropic, inhomogeneous materials. Subsequently, we derived a Bessel-Fourier expansion of the scattered electric field via point-matching of the FEM solution on a near-zone circle, and from there computed the 2-D *bistatic* radar cross-section [10] (RCS) $\sigma_{2D}(\phi,\omega)$, with $\phi = \arctan(y/x)$. The FEM simulations below refer to a $14\lambda_0 \times 14\lambda_0$ computational domain (λ_0 being the free-space wavelength), discretized into $\sim$800000 unknowns. Fig. 3(a) illustrates, for a structure with a moderate electric width $2L = 6\sqrt{2}\lambda_0$ and thickness $h = L/(3\sqrt{2}) = \lambda_0$, a representative near-field map for oblique incidence with $\phi_i = 15°$. Also shown, as references, are the corresponding responses of a corner reflector with identical aperture [Fig. 3(b)] and a plain PEC sheet of same width [Fig. 3(c)]. The retrodirective response of the proposed reflector is clearly visible from the wavefront shapes in Fig. 3(a), in complete accord with those observed in the corner-reflector case [Fig. 3(b)], and in sharp contrast with those observed in the plain-PEC-sheet case [Fig. 3(c)]. This is even more evident in the bistatic RCS responses shown in Fig. 4, where, besides the obvious forward-direction peaks, a retrodirective $(\phi = -\phi_i)$ peak is observed for the proposed and corner reflectors [Fig. 4(a)], whereas a specular $(\phi = \phi_i)$ peak is observed for the plain PEC sheet [Fig. 4(b)].

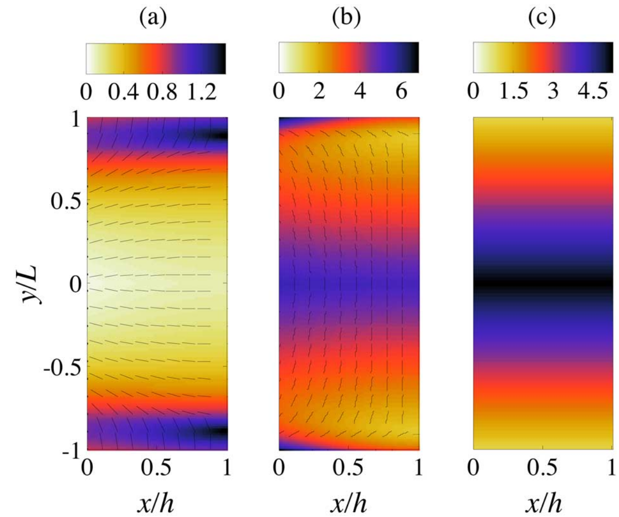

Fig. 2. (Color online) Spatial distribution of the material parameters in (3), for a coating thickness $h = L/(3\sqrt{2})$, in the (u,v,z) principal reference system. (a), (b), and (c): u, v, and z components, respectively. As a reference, the local directions of the principal axes u and v are shown as short segments in (a) and (b), respectively.

Note that the moderate thickness value $h = L/(3\sqrt{2}) = \lambda_0$ in the above example was only chosen in order to allow a better visualization of the internal field, and consistently similar responses were observed for thinner slabs up to $h = L/(24\sqrt{2}) = \lambda_0/8$, with only slight differences attributable to the different meshing in the FEM solver. Fig. 5 compares the retrodirective response [in terms of the *monostatic* RCS $\sigma_{2D}(-\phi_i,\omega)$] of the proposed reflector with a reduced thickness of $h = L/(24\sqrt{2}) = \lambda_0/8$ to those of the reference corner reflector (practically indistinguishable on the scale of the plot) and plain PEC sheet, as a function of the incidence angle ϕ_i. As expectable, for incidence close to normal, the retrodirective response of the proposed reflector is comparable with that of the plain PEC sheet, but becomes comparatively much stronger for oblique incidence, though decreasing for larger values of the incidence angle (as typical of corner reflectors [10]). Also shown are the results pertaining to *lossy* metamaterial implementations. As typically observed in other transformation-optics applications, the response deterioration

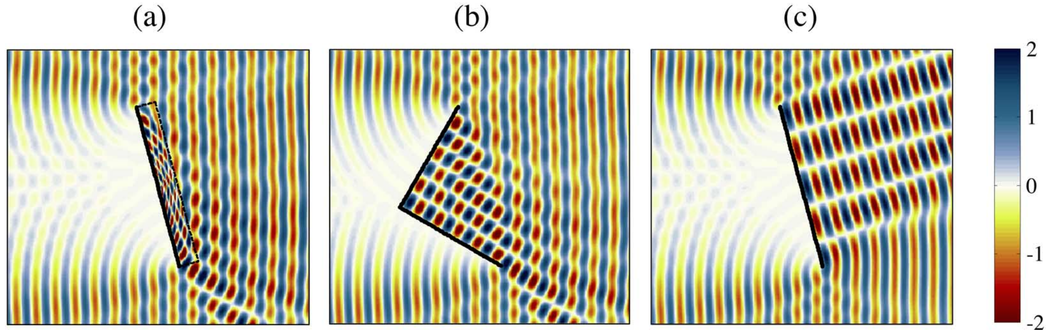

Fig. 3. (Color online) FEM-computed near-field maps (real part of electric field) for unit-amplitude plane-wave oblique ($\phi_i = 15^\circ$) incidence. (a): Proposed reflector with $2L = 6\sqrt{2}\lambda_0$ and $h = L/(3\sqrt{2}) = \lambda_0$, and lossless material parameters. (b): Corner-reflector of same aperture. (c): Plain PEC sheet of same width. For computational convenience, the structures are rotated and the illuminating wave is impinging horizontally (from right).

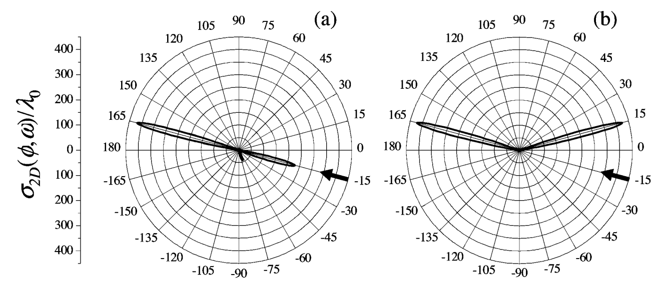

Fig. 4. As in Fig. 3, but 2-D bistatic RCS (scaled to the free-space wavelength). (a) Proposed and corner reflectors (continuous and dashed curve, respectively—practically indistinguishable on the scale of the plot). (b) Plain PEC sheet. The incidence direction is marked by a thick arrow.

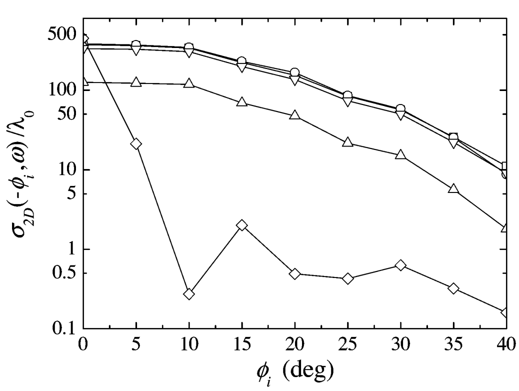

Fig. 5. As in Fig. 4, but monostatic RCS as a function of the incidence angle ϕ_i, for the proposed reflector with reduced coating thickness $h = L/(24\sqrt{2}) = \lambda_0/8$ (squares), and for the reference corner reflector (circles) and plain PEC sheet (diamonds). Also shown, as references, are the results pertaining to the proposed reflector with lossy materials characterized by a loss-tangent of 0.001 (down-triangles) and 0.01 (up-triangles).

is rather mild for low losses $(\text{loss}-\text{tangent} = 0.001)$, and becomes more visible, though not dramatic, for moderate losses $(\text{loss}-\text{tangent} = 0.01)$.

III. Conclusion

To sum up, we have shown that thin planar retrodirective reflectors can be effectively designed by using corner-reflector-inspired transformation media. With the current trends of metamaterial technology, the practical feasibility of the involved materials appears within reach, so that the results of our prototype study could suggest a viable and attractive alternative to traditional circuit-based (e.g., Van Atta or heterodyne-type) approaches. In this connection, it should be emphasized that important applications of interest, such as identification, are intrinsically *narrowband*, and, therefore, the severe bandwidth restrictions arising from a practical metamaterial realization should not be a major concern.

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References

[1] D. Schurig, J. B. Pendry, and D. R. Smith, “Calculation of material properties and ray tracing in transformation media,” *Opt. Express*, vol. 14, no. 21, pp. 9794–9804, Oct. 2006.
[2] U. Leonhardt and T. G. Philbin, “General relativity in electrical engineering,” *New J. Phys.*, vol. 8, no. 10, 247, Oct. 2006.
[3] D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, “Metamaterial electromagnetic cloak at microwave frequencies,” *Science*, vol. 314, no. 5801, pp. 977–980, Nov. 2006.
[4] A. Greenleaf, Y. Kurylev, M. Lassas, and G. Uhlmann, “Electromagnetic wormholes and virtual magnetic monopoles from metamaterials,” *Phys. Rev. Lett.*, vol. 99, no. 18, 183901, Nov. 2007.
[5] H. Y. Chen and C. T. Chan, “Transformation media that rotate electromagnetic fields,” *Appl. Phys. Lett.*, vol. 90, no. 24, 241105, Jun. 2007.
[6] O. Ozgun and M. Kuzuoglu, “Electromagnetic metamorphosis: Reshaping scatterers via conformal anisotropic metamaterial coatings,” *Microw. Opt. Technol. Lett.*, vol. 49, no. 10, pp. 2386–2392, Oct. 2007.
[7] B. Donderici and F. L. Teixeira, “Metamaterial blueprints for reflectionless waveguide bends,” *IEEE Microw. Wireless Compon. Lett.*, vol. 18, no. 4, pp. 233–235, Apr. 2008.
[8] F. Kong, B.-I. Wu, J. A. Kong, J. Huangfu, S. Xi, and H. Chen, “Planar focusing antenna design by using coordinate transformation technology,” *Appl. Phys. Lett.*, vol. 91, no. 25, 253509, Dec. 2007.
[9] M. Tsang and D. Psaltis, “Magnifying perfect lens and superlens design by coordinate transformation,” *Phys. Rev. B*, vol. 77, no. 3, 035122, Jan. 2008.
[10] E. F. Knott, J. F. Shaeffer, and M. T. Tuley, *Radar Cross Section*, 2nd ed. London, U.K.: Artech House, 1993.
[11] Y. Zhang, B. Fluegel, and A. Mascarenhas, “Total negative refraction in real crystals for ballistic electrons and light,” *Phys. Rev. Lett.*, vol. 91, no. 15, 157404, Oct. 2003.
[12] W. Cai, U. K. Chettiar, A. V. Kildishev, V. M. Shalaev, and G. W. Milton, “Nonmagnetic cloak with minimized scattering,” *Appl. Phys. Lett.*, vol. 91, no. 11, 111105, Sep. 2007.
[13] COMSOL MULTIPHYSICS—User’s Guide 2005, COMSOL AB.