# Radiation Properties of One-Dimensional Random-Like Antenna Arrays Based on Rudin–Shapiro Sequences

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***Abstract*—The development of exotic new materials, such as metamaterials, has created strong interest within the electromagnetics (EM) community for possible new phenomenologies and device applications, with particular attention to periodicity-induced phenomena, such as photonic bandgaps. Within this context, motivated by the fairly recent discovery in X-ray crystallography of "quasi-crystals," whose diffraction patterns display unusual characteristics that are associated with "aperiodic order," we have undertaken a systematic study of how these exotic effects manifest themselves in the radiation properties of aperiodically configured antenna arrays. The background for these studies, with promising example configurations, has been reported in a previous publication [V. Pierro *et al.*, *IEEE Trans. Antennas Propag.*, vol. 53, pp. 635–644, Feb. 2005]. In this paper, we pay attention to various configurations generated by Rudin–Shapiro (RS) sequences, which constitute one of the simplest conceivable examples of *deterministic aperiodic* geometries featuring *random-like* (dis)order. After presentation and review of relevant background material, the radiation properties of one-dimensional RS-based antenna arrays are analyzed, followed by illustrative numerical parametric studies to validate the theoretical models. Design parameters and potential practical applications are also given attention.**

***Index Terms*—Antenna array, aperiodic order, random-like geometries, Rudin–Shapiro sequences.**

## I. Introduction

In AN ongoing plan of investigations [1], [2], we have recently been concerned with the study of the radiation properties of structures with *aperiodic order*. Interest in this type of geometries is physically motivated by the recent discovery of "quasi-crystals" [3], [4], i.e., certain metallic alloys displaying X-ray diffraction properties that are known to be *incompatible* with spatial periodicity. In electromagnetic (EM) engineering, this has so far produced interesting applications in the field of photonic bandgap "quasi-crystal" devices (see the brief summary and references in [1]).

We have initiated our series of prototype studies with the aim of exploring the radiation properties of selected categories of aperiodic geometries, representative of the various hierachial types and degrees of aperiodic order in the "gray zone" that separates *perfect periodicity* from *absolute randomness*. To this end, we started in [1] with exploring the basic radiation properties of two-dimensional (2-D) antenna arrays based on certain representative categories of "aperiodic tilings" (see [5] and [6] for an introduction to the subject). In parallel, we also began investigating the radiation properties of selected categories of one-dimensional (1-D) *aperiodic sequences,* which are more easily amenable to explicit *analytic parameterization* and can thus provide better insight into the relevant wave-dynamical phenomenologies. Within this framework, we explored in [2] the radiation properties of 1-D *quasi-periodic* antenna arrays based on the *Fibonacci sequence,* which admit analytic parameterization in terms of *generalized Poisson summation* formulas. In this paper, we proceed along the same route but focus on the *opposite* end of the above-mentioned aperiodic-order "gray zone," namely, the *random-like* type of (dis)order. One of the simplest and most intriguing examples is provided by the *Rudin–Shapiro (RS) sequences* [7]–[9]. These sequences were developed in a pure-mathematics context by Shapiro and Rudin during the 1950s, pertaining to some extremal problems in harmonic analysis. More recently, RS sequences have found important practical applications as signal processing tools for *spread spectrum* communication and encryption systems [10], [11]. Moreover, the recent advances in materials engineering have motivated a growing interest toward the analysis of the (EM, acoustic, elastic) wave-dynamical properties of RS-based structures. In this framework, available studies [12]–[21] are principally focused on the electron (or phonon) transport and localization properties in RS-based lattices, as well as wave propagation in RS-based multilayers. These studies, often based on *tight-binding* models and *trace-map* formalisms, have revealed intriguing properties in terms of spectral scaling, as well as eigenfunction localization and rich self-similar structure, which turn out to be *intermediate* between those of quasi-periodic and disordered structures. The reader is referred to [12]–[21] and the references therein for more details.

In line with our stated interest in the radiation properties of aperiodically parameterized antenna arrays, we concentrate in this paper on 1-D RS-based antenna array configurations. Our implementation via analytic and numerical studies makes use of well-established spectral and statistical properties of RS sequences.

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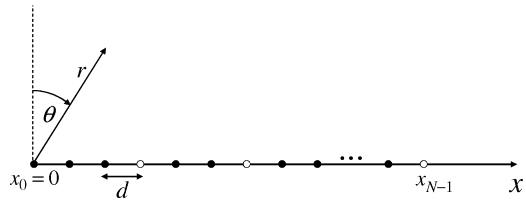


Fig. 1. Problem geometry: a 1-D array of $N$ identical antenna elements with uniform interelement spacing $d$ is considered, with antenna element positions at $x_n = nd, n = 0, \ldots, N-1$, and uniform interelement phasing $\eta$. Black and white dots denote the two possible values of the real excitation amplitudes $I_n$ in (1), related to the RS sequence (see Section II-B). Also shown is the array-end-centered polar $(r, \theta)$ coordinate system.

Accordingly, the remainder of this paper is laid out as follows. Section II contains the problem formulation, with description of the geometry and review of the basic properties of RS sequences. Section III deals with the radiation properties of representative classes of RS-based antenna arrays; in this connection, potential applications to *omnidirectional* and *thinned* arrays are proposed and discussed. Conclusions are provided in Section IV.

## II. Background and Problem Formulations

### A. Geometry

The problem geometry, a 1-D array of $N$ identical antenna elements with uniform interelement spacing $d$, is sketched in Fig. 1. Without loss of generality, the array is assumed to be placed on the $x$ axis, with radiators at $x_n = nd, n = 0, \ldots, N-1$; in view of the rotational symmetry around the $x$ axis, we adopt the 2-D polar $(r, \theta)$ coordinate system. Neglecting interelement coupling, and for an implied $\exp(j\omega t)$ time dependence, the $n$th antenna element complex excitation is denoted by

$$\tilde{I}_n = I_n \exp(-jk_0 nd\eta) \tag{1}$$

where $I_n$ is a real excitation amplitude, which, by assumption, can take on only two values (denoted by black and white dots in Fig. 1) related to the RS sequence described in Section II-B. Moreover, $k_0 = \omega\sqrt{\epsilon_0\mu_0} = 2\pi/\lambda_0$ is the free-space wavenumber (with $\lambda_0$ being the wavelength) and $\eta$ describes the interelement phasing. For observation distances $r \gg 2(Nd)^2/\lambda_0$ (Fraunhofer region), the array is characterized by the "array factor" [22]

$$\begin{aligned} F_s(\xi) &= \sum_{n=0}^{N-1} I_n \exp(j2\pi n\xi) \\ &= \sum_{n=0}^{N-1} I_n \exp\left[jk_0 nd(\sin\theta - \eta)\right] = F_p(\theta) \end{aligned} \tag{2}$$

where $\xi = d(\sin\theta - \eta)/\lambda_0$ is a nondimensional angular spectral variable. In what follows, we shall refer to $|F_s(\xi)|^2$ as the "radiation spectrum" and to $|F_p(\theta)|^2$ as the "radiation pattern." The 1-D results discussed here can be extended to 2-D geometry-separable configurations via conventional pattern multiplication [22].

### B. Rudin–Shapiro Sequences

RS sequences are two-symbol aperiodic sequences with *random-like* character [7]–[9]. In what follows, their basic properties are briefly reviewed, with emphasis on those aspects that are directly relevant to the antenna array application in Section III (see [23]–[27] for more details). In its basic form, a RS sequence is generated from the *two-symbol alphabet* $\mathcal{A}_a = \{-1, 1\}$ via the simple recursive rule

$$\alpha_0 = 1, \quad \alpha_{2n} = \alpha_n, \quad \alpha_{2n+1} = (-1)^n \alpha_n. \tag{3}$$

Thus, for instance, the first ten symbols in the sequence are

$$1\,1\,1\,-1\,1\,1\,-1\,1\,1\,1. \tag{4}$$

It can be shown that, in the limit of an *infinite* sequence, the two symbols have the same statistical frequency of occurrence [26]. Throughout this paper, the sequence $\{\alpha_n\}$ in (3) will be referred to as an *alternate* RS (ARS) sequence. This sequence can also be generated via a two-step procedure involving a substitution rule defined on a four-symbol alphabet $\mathcal{A}_s = \{A, B, C, D\}$

$$A \to AB, \quad B \to AC, \quad C \to DB, \quad D \to DC \tag{5a}$$

and final projections $\Pi : \mathcal{A}_s \to \mathcal{A}_a$

$$\Pi(A) = 1, \ \Pi(B) = 1, \ \Pi(C) = -1, \ \Pi(D) = -1. \tag{5b}$$

Another possible construction algorithm is based on the use of the so-called *RS polynomials* [11], [25], [28] defined by the recursive rules

$$P_{m+1}(\xi) = P_m(\xi) + \exp(j2\pi 2^m \xi) Q_m(\xi), \ P_0 = 1 \tag{6a}$$
$$Q_{m+1}(\xi) = P_m(\xi) - \exp(j2\pi 2^m \xi) Q_m(\xi), \ Q_0 = 1. \tag{6b}$$

It can be verified that $P_m$ and $Q_m$ in (6) are $(2^m - 1)$th degree trigonometric polynomials in the variable $\exp(j2\pi\xi)$

$$P_m(\xi) = \sum_{n=0}^{2^m-1} p_n \exp(j2\pi n\xi) \tag{7a}$$
$$Q_m(\xi) = \sum_{n=0}^{2^m-1} q_n \exp(j2\pi n\xi). \tag{7b}$$

The link between RS polynomials and sequences stems from the polynomial coefficients $p_n$ and $q_n$ in (7). For $P_m$-type polynomials, the coefficient sequence coincides with the ARS sequence, i.e., $p_n = \alpha_n, n = 0, \ldots, 2^m - 1$. For $Q_m$-type polynomials, the coefficient sequence (technically defined as ARS Golay-complementary [29]) is obtained in practice by reversing the sign of the second half $(n \geq 2^{m-1})$ of the ARS sequence

$$q_n = \begin{cases} \alpha_n, & 0 \leq n \leq 2^{m-1} - 1 \\ -\alpha_n, & 2^{m-1} \leq n \leq 2^m - 1 \end{cases}. \tag{8}$$

The reader is referred to [25], [30], and [31] for more details and possible generalizations of ARS sequences. Antenna arrays based on ARS sequences are treated in Section III-A.

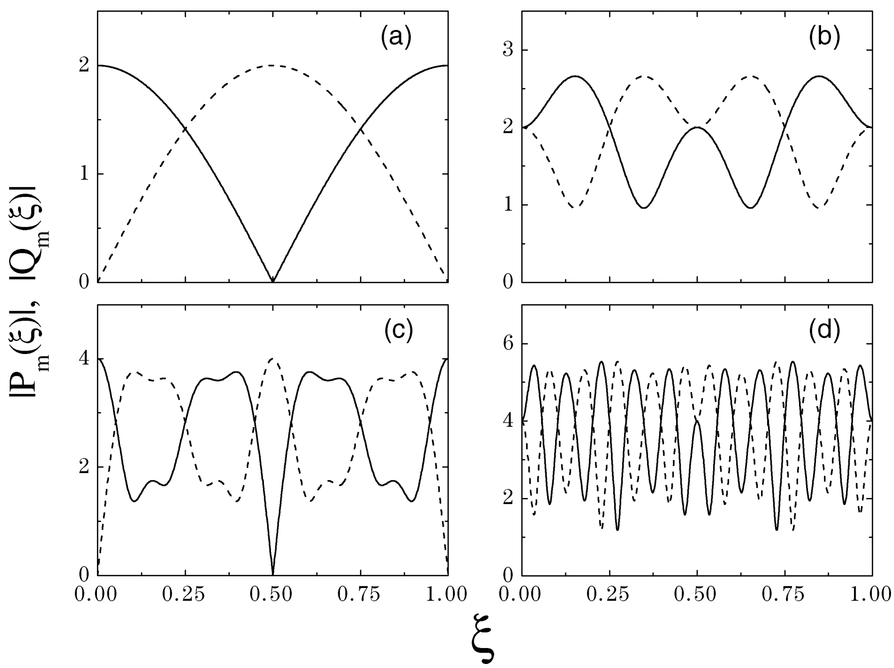


Fig. 2. RS polynomials $P_m$ (solid curves) and $Q_m$ (dashed curves) for various orders $m$. (a)–(d) $m = 1, 2, 3, 4$, respectively (see Section II-B for details).

Another RS sequence of interest for this investigation (see Section III-B) is related to the ARS sequence in (3) via the simple linear transformation

$$\beta_n = \frac{1-\alpha_n}{2}. \tag{9}$$

It is readily verified that the sequence $\{\beta_n\}$ in (9) is composed of symbols from the alphabet $\mathcal{A}_b = \{0, 1\}$, and will accordingly be defined as *binary* RS (BRS) sequence.

Despite the *fully deterministic* and *deceptively simple* character of (3) and (9), RS sequences exhibit some *statistical and spectral* properties that are remarkably different from those of typical deterministic sequences, and similar in some respects to those of *random* sequences. In particular, the correlation properties of the ARS sequence in (3) resemble those of white-noise-like random sequences [27]

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} \alpha_n \alpha_{n+\nu} = \delta_{\nu 0} \tag{10}$$

where $\delta_{pq}$ is the Kronecker delta symbol, $\delta_{pq} = 1$ for $p = q$, $\delta_{pq} = 0$ otherwise.

From the spectral viewpoint, ARS sequences exhibit interesting *extremal* properties. It can be proved that their truncated Fourier series amplitude is bounded as follows [8]:

$$\sqrt{N} \leq \sup_{0\leq\xi<1} \left| \sum_{n=0}^{N-1} \alpha_n \exp(j2\pi n\xi) \right| \leq (2+\sqrt{2})\sqrt{N}. \tag{11}$$

Note that while the lower bound in (11) holds for *any* sequence $\{\alpha_n\}$ generated from the alphabet $\mathcal{A}_a = \{-1, 1\}$ [27], the upper bound is peculiar of ARS sequences. For a general sequence $\{\alpha_n\}$ generated from the alphabet $\mathcal{A}_a$, the upper bound in (11) would be $N$ (with the equality sign holding, e.g., for *periodic* sequences) [23], [27]. On the other hand, for *random* sequences in $\mathcal{A}_a$, the upper bound typically behaves like $\sqrt{N \log N}$ [23].

From (11), it can be proved that the Fourier spectrum of the ARS sequence tends to a *constant* value in the infinite sequence limit [23], and is thus devoid of *any* strongly localized spectral footprints. Again, this behavior resembles the "flat-spectrum" character of white-noise-like random sequences. For the RS polynomials in (6) and (7), a behavior similar to (11) is found, with a *tighter* upper bound [11]

$$|P_m(\xi)| \leq 2^{\frac{m+1}{2}}, \quad |Q_m(\xi)| \leq 2^{\frac{m+1}{2}} \tag{12}$$

[compare with (11), with $N = 2^m$]. The behavior of $P_m$ and $Q_m$ RS polynomials in a single period $0 \leq \xi < 1$ is shown in Fig. 2, for $1 \leq m \leq 4$. It is observed that, at any order, the two polynomial types exhibit *complementary* behaviors (maxima of $P_m$ correspond to minima of $Q_m$, and vice versa). Moreover, all the odd-order $(m = 2l + 1)$ RS polynomials have their only zeros at $\xi = 1/2$ ($P_m$-type) and $\xi = 0, 1$ ($Q_m$-type) [see Fig. 2(a) and (c)], whereas even-order $(m = 2l)$ polynomials do not vanish for $0 \leq \xi < 1$ [see Fig. 2(b) and (d)]. Actually, it can be shown that, in general [11]

$$P_{2l}(0) = P_{2l}\left(\frac{1}{2}\right) = 2^l$$

$$P_{2l+1}(0) = 2^{l+1}, \quad P_{2l+1}\left(\frac{1}{2}\right) = 0 \tag{13a}$$

$$Q_{2l}(0) = -Q_{2l}\left(\frac{1}{2}\right) = 2^l$$

$$Q_{2l+1}(0) = 0, \quad Q_{2l+1}\left(\frac{1}{2}\right) = 2^{l+1}. \tag{13b}$$

Concerning the BRS sequence in (9), its Fourier spectrum results from the superposition of a *flat* background and a *periodic* distribution ($\xi \in \mathbb{Z}$) of spectral peaks [see [23] and [32] for details]. This could be expected by noting that the BRS sequence in (9) can be interpreted as the superposition of an ARS sequence and a *periodic* sequence, whose cross-correlation can be proved to be zero [26], [27]. Quite remarkably, the BRS sequence exhibits *the same* Fourier spectrum as a Bernoulli *random* sequence in $\mathcal{A}_b = \{0, 1\}$ whose symbols are chosen with equal probability of occurrence [32], [33]; these two sequences are said to be *homometric* [32] in technical jargon.

## III. Rudin–Shapiro Antenna Arrays

We now investigate in detail the radiation properties of representative classes of antenna arrays based on RS sequences. Referring to the geometry in Fig. 1, the RS character is embedded in the array via the excitation amplitudes $I_n$ in (2). Two cases are considered: a) ARS sequence ($I_n = \alpha_n$) and b) BRS sequence ($I_n = \beta_n$). Parametric studies have been performed, with emphasis on two meaningful array-oriented observables: the maximum directivity $D_M$ and the side-lobe ratio (SLR) $\rho_S$, which can be compactly defined as

$$D_M = \frac{2\|F_p\|_\infty^2}{\|F_p\|_2^2} \tag{14}$$

$$\rho_S = \frac{\sup_{\theta \notin \mathcal{B}} |F_p(\theta)|^2}{\|F_p\|_\infty^2}. \tag{15}$$

In (14) and (15), $\|\cdot\|_\infty$ and $\|\cdot\|_2$ denote the $L_\infty$ (supremum) and $L_2$ [root mean square (rms)] norms, respectively

$$\|F_p\|_\infty^2 = \sup_{-\frac{\pi}{2} \leq \theta < \frac{\pi}{2}} |F_p(\theta)|^2$$

$$\|F_p\|_2^2 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |F_p(\theta)|^2 \cos\theta d\theta \tag{16}$$

and $\mathcal{B}$ denotes the main lobe angular region. Using (2), one can show that

$$\|F_p\|_2^2 = 2c_0 + 4\sum_{\nu=1}^{N-1} c_\nu \frac{\cos(k_0 \nu d \eta)\sin(k_0 \nu d)}{k_0 \nu d} \tag{17a}$$

where the $c_\nu$ correlation sums

$$c_\nu = \sum_{l=0}^{N-1-\nu} I_l I_{l+\nu}. \tag{17b}$$

This provides the background for the theoretical considerations and synthetic numerical experiments that follow.

### A. Flat-Spectrum "Omnidirectional" Arrays

We begin with the class of antenna arrays based on the *ARS sequence* $\{\alpha_n\}$ in (3). A typical example of the radiation spectrum is shown in Fig. 3 for a 100-element array. As could be expected from the spectral properties of the ARS sequence discussed in Section II-B, the radiation spectrum exhibits a fairly *flat* behavior (no distinct localized spectral peaks) with truncation-induced rapid oscillations. This results in a radiation pattern with globally "omnidirectional" characteristics. Interestingly, for this class of arrays, one can establish some general theoretical bounds on the maximum directivity $D_M$, which are practically independent of the interelement spacing and phasing. First, one can show that the radiation pattern satisfies the constraint

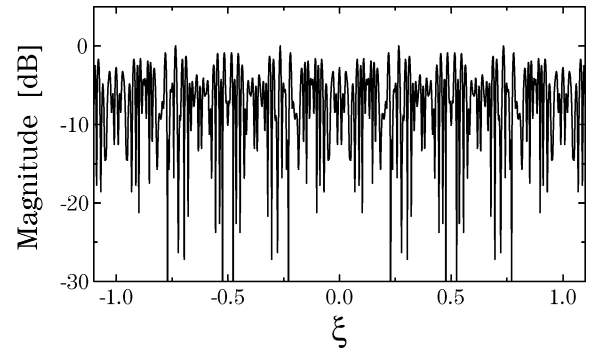


Fig. 3. ARS antenna array with $N = 100$ elements. Radiation spectrum $|F_s(\xi)|^2$ (normalized to its maximum value) versus spectral parameter $\xi$ [see (2)]. Maximum directivity is $D_M \approx 5.7$ dB, almost independent of the interelement spacing and phasing.

$$\|F_p\|_\infty^2 \leq (2+\sqrt{2})^2 N, \quad \|F_p\|_2^2 \approx 2c_0 = 2N. \tag{18}$$

In (18), the upper bound on the $L_\infty$ norm follows directly from (11). The approximation for the $L_2$ norm, numerically verified a posteriori (see below), is obtained from (17a) by neglecting the $\nu$-summation; this term vanishes exactly for $d = p\lambda_0/2$, $p \in \mathbb{N}$, and is more generally negligible since—in view of (10) and (17b), for arrays of *finite* but moderate to large size—one expects $|c_\nu| \ll c_0$, for $\nu \geq 1$. Accordingly, it follows from (14) that the bound on the maximum directivity $D_M$ is given by

$$D_M \lesssim (2+\sqrt{2})^2 \approx 10.7 \text{ dB}. \tag{19}$$

By comparison with the computed value [via (14)] in Fig. 3 (which is $D_M \approx 5.7$, practically independent of $d/\lambda_0$ and $\eta$), this *general* bound turns out to be rather *loose*. In this connection, much tighter bounds can be established for arrays based on the RS polynomials in (6). In view of (7), these arrays can be synthesized from the array factor $F_s(\xi)$ in (2) with $I_n = p_n$, $q_n$, and $N = 2^m$, yielding radiation patterns of the type

$$|F_p(\theta)|^2 = \begin{cases} \left|P_m\left[\frac{d(\sin\theta-\eta)}{\lambda_0}\right]\right|^2 \\ \left|Q_m\left[\frac{d(\sin\theta-\eta)}{\lambda_0}\right]\right|^2 \end{cases}. \tag{20}$$

For this class of arrays, the relationships in (18) can be rewritten as

$$\|F_p\|_\infty^2 \leq 2^{m+1}, \quad \|F_p\|_2^2 \approx 2 \cdot 2^m \tag{21}$$

where the upper bound on the $L_\infty$ norm now follows from (12), whereas the approximation for the $L_2$ norm is obtained and justified as before [see (18), with $N = 2^m$]. This yields the following bound on the maximum directivity

$$D_M \lesssim 2 \approx 3 \text{ dB} \tag{22}$$

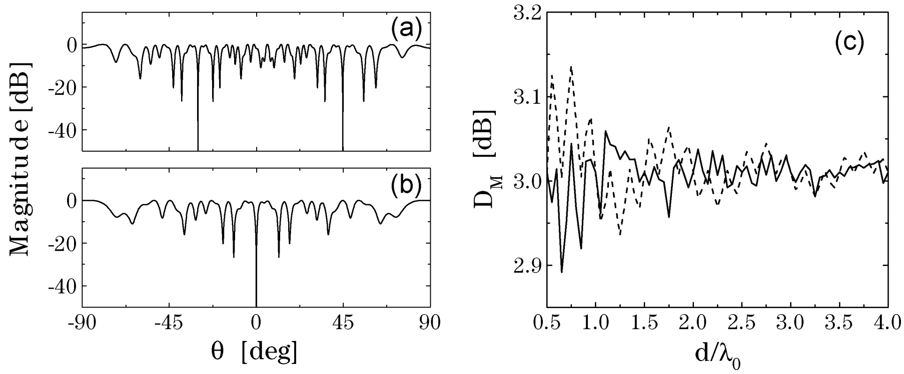


Fig. 4. Examples of antenna arrays synthesized using the RS polynomials $P_5$ and $Q_5$ ($N = 32$ elements). (a) Radiation pattern $|F_p(\theta)|^2$ (normalized to its maximum value) for $P_5$-type array, with $d = 0.83\lambda_0$ and $\eta = 0.1$ (with prescribed nulls at $\theta = -30^\circ$ and $\theta = 45^\circ$), having maximum directivity $D_M = 2.9$ dB. (b) Radiation pattern for $Q_5$-type array, with $d = 0.5\lambda_0$ and $\eta = 0$ (with prescribed null at $\theta = 0$), having maximum directivity $D_M = 3$ dB. (c) Maximum directivity $D_M$ versus interelement spacing for the two configurations in (a) and (b), shown as solid and dashed curves, respectively.

which is consequently *tighter* than the general bound in (19). By this synthesis, one could therefore obtain an antenna array with nearly "omnidirectional" characteristics. Moreover, for the special case of odd-order $(m = 2l + 1)$ RS polynomials, by exploiting the two available degrees of freedom ($d/\lambda_0$ and $\eta$) to scale and shift the spectral zeros in (13), one can place up to two nulls at arbitrary directions, while keeping the *global* omnidirectional character. Two examples are illustrated in Fig. 4, in connection with fifth-order RS polynomials ($N = 32$ antenna elements). Specifically, Fig. 4(a) shows the radiation pattern synthesized with a $P_5$ polynomial, with array parameters ($d/\lambda_0 = 0.83$ and $\eta = 0.1$) chosen so as to place two nulls at $\theta = -30^o$ and $\theta = 45^o$. Fig. 4(b) shows the radiation pattern synthesized with a $Q_5$ polynomial, with array parameters ($d/\lambda_0 = 0.5$ and $\eta = 0$) chosen so as to place a null at broadside ($\theta = 0$). The maximum directivity versus the interelement spacing is shown in Fig. 4(c) for both configurations. An essentially *flat* behavior (magnified by the expanded scale chosen in the plot) is observed, with computed values very close to the theoretical bound in (22); this also serves as an indirect a posteriori validation for the approximation of the $L_2$ norm used in (18) and (21). Clearly, in view of the omnidirectional character, with the absence of a main radiation beam, the SLR parameter $\rho_S$ in (15) is not of interest for this class of arrays.

### B. Thinned Arrays

Next, we move on to antenna arrays based on the *BRS sequence* $\{\beta_n\}$ in (9). Assuming $I_n = \beta_n$ in the array factor in (2), and recalling that $\beta_n \in \mathcal{A}_b = \{0, 1\}$ (i.e., antenna elements are either "off" or "on"), one may interpret this choice as a special *deterministic* "array thinning" strategy [22]. Recalling the spectral properties of BRS sequences summarized at the end of Section II-B, one can expect a radiation spectrum constituted of an ARS-type *flat* background (see Fig. 3) with superposed *discrete* spectral peaks (grating lobes) typical of *periodic* arrays. These features are clearly visible in the example shown in Fig. 5 for an array with $N = 200$ featuring $N_a = 90$ "active" (i.e., $I_n = 1$) elements. Based on these premises, and recalling the symbol equioccurrence property for infinite RS sequences, one may anticipate radiation properties (in terms of beamwidth, directivity, and SLR) comparable to those of a fully

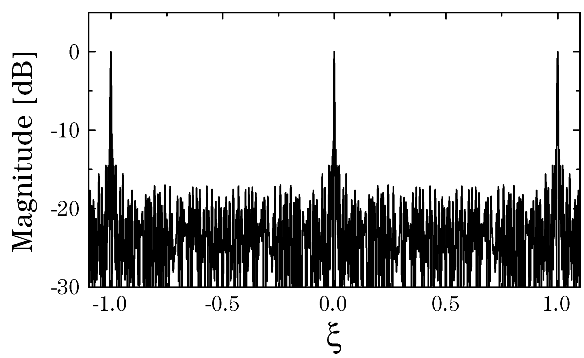


Fig. 5. Thinned BRS antenna array with $N = 200$ ($N_a = 90$ active elements). Radiation spectrum $|F_s(\xi)|^2$ (normalized to its maximum value) versus spectral parameter $\xi$.

populated *periodic array,* using *nearly half* of the elements, at the expense of a slight pattern degradation (due to the flat background spectrum), which is, however, expected (and acceptable) in most typical array-thinning strategies [22]. In other words, for a *fixed* number of active antenna elements $N_a$ and array aperture $L_A$ (defined as the distance between the first and last active elements), BRS-type arrays could yield an increase by a factor $\sim 2$ in the maximum value allowable for the average interelement spacing $d_{av} = L_A/(N_a - 1)$ so as to avoid the appearance, in the visible range, of grating lobes ($d_{av} \sim 2\lambda_0$ versus $d_{av} = \lambda_0$ for periodic arrays).

Fig. 6 illustrates the radiation characteristics of a typical BRS-type array with $N_a = 90$ active elements. Specifically, Fig. 6(a) and (b) shows the radiation patterns for average interelement spacings $d_{av} = \lambda_0$ and $d_{av} = 2\lambda_0$, respectively. Also shown, for comparison, is the reference behavior of a periodic array with same number of elements and interelement spacing (and, hence, comparable beamwidth). As expected, while the periodic array displays the well-known grating lobes, the BRS array *does not.* The expected slight degradation in the radiation pattern (due to the flat background spectrum) is observed with amplitude below the secondary lobe level ($\sim -13$ dB). This is better quantified in Fig. 6(c) and (d), where the maximum directivity $D_M$ and the SLR $\rho_S$ versus the average interelement spacing are shown. It is observed that the BRS-array characteristics are only slightly worse than those of the periodic array

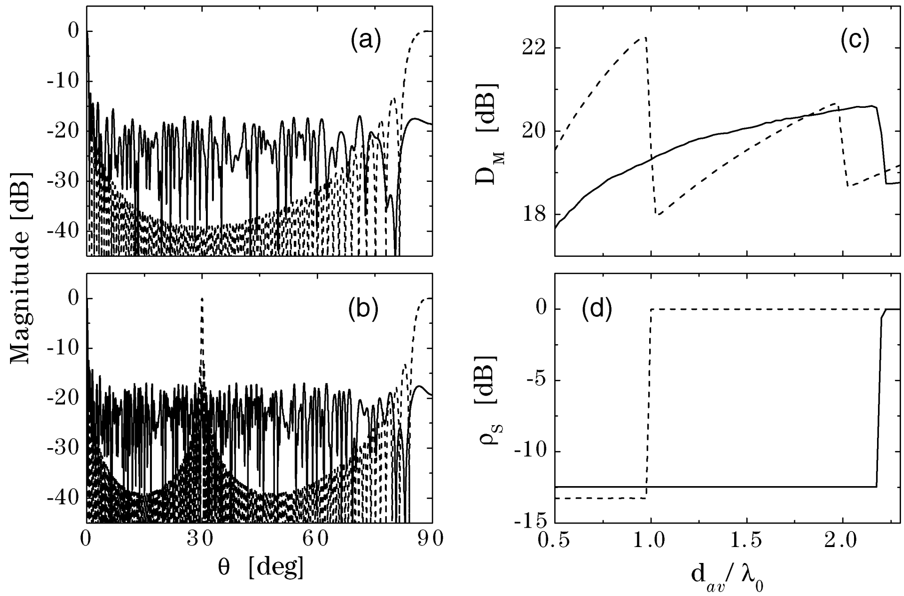


Fig. 6. As in Fig. 5, but radiation characteristics for $\eta = 0$ (solid curves). (a), (b) Radiation patterns $|F_p(\theta)|^2$ (normalized to their maximum value) for average interelement spacings $d_{av} = \lambda_0$ and $d_{av} = 2\lambda_0$, respectively. (c), (d) Maximum directivity $D_M$ in (14) and side-lobe ratio $\rho_S$ in (15) versus average interelement spacing. Also shown (dashed), for comparison, is the reference behavior of a periodic array of the same size.

TABLE I
BRS Antenna Array as in Fig. 6. Maximum Directivity $D_M$ and SLR $\rho_S$ for $d_{av} = \lambda_0$, $\eta = 0$, and Various Values of the Number of Active Elements $N_a$. The SLR Values Obtained From Numerical Simulations Were Almost Independent of $d_{av}$ and $\eta$

| $N_a$ | $D_M$ [dB] | $\rho_s$ [dB] |
|---|---|---|
| 10 | 9.67 | -5.9 |
| 25 | 13.6 | -8.8 |
| 50 | 16.8 | -11.9 |
| 100 | 19.8 | -11.2 |
| 250 | 23.8 | -13.8 |
| 500 | 26.9 | -12.3 |

for $d_{av} < \lambda_0$ and become almost uniformly better beyond the critical value $d_{av} = \lambda_0$, at which the periodic array characteristics experience an abrupt degradation due to the entrance in the visible range of the first grating lobe [see Fig. 6(a)]. This trend holds up to values of $d_{av} \approx 2.2\lambda_0$, at which the first grating lobe of the BRS array enters the visible range. Qualitatively similar behaviors have been observed by us for arrays of different size. Some representative results are summarized in Table I in terms of maximum directivity and SLR for the case $d_{av} = \lambda_0$ and $\eta = 0$. It is observed that, with the exception of small-size arrays ($N_a \lesssim 50$), the SLR values are always comparable with the reference periodic-array level ($\sim -13$ dB).

## IV. Conclusions and Perspectives

This paper has dealt with the radiation properties of 1-D random-like antenna arrays based on RS sequences. Potential applications have been suggested, based on known spectral and statistical properties of these sequences. Results so far seem to indicate that RS sequences might provide new perspectives for the synthesis of antenna arrays.

In particular, the ARS-type synthesis described in Section III-A might be of potential interest for smart reconfigurable antenna systems where *omnidirectional* receiving/transmitting features are needed, as a possible alternative to the code-based synthesis approaches in [34]. Other potentially interesting applications could be envisaged in the area of reflectarray synthesis of "simulated corrugated surfaces" [35] and, more generally, "virtual shaping" applications in radar countermeasures [36]. In this connection, use could be made of recent advances in the practical synthesis of artificial impedance surfaces [37] for designing 1-D (strip) or 2-D (patch) planar arrangements based on suitable "dual" impedance surfaces. Examples are perfect-electric and perfect-magnetic conductors capable of providing the opposite-sign reflection coefficients, which, in a first-order physical optics approximation of the relevant scattering problem, play the role of the ARS excitation amplitudes in the array factor here. Such structures, currently under investigation, could yield strong reduction of specular reflection and *uniform* angular distribution of the scattered power.

On the other hand, the BRS-type synthesis described in Section III-B seems to offer a simple *fully deterministic* alternative to typical *random* thinning strategies [22] for arrays of moderate to large size. In this connection, it is interesting to observe that, in view of the *homometry* properties mentioned at the end of Section III, this strategy is in a sense *equivalent* to a *completely random* scheme, in which the antenna elements are turned on/off with equal probability.

Planned future investigations involve the extension to 2-D geometries, as well as applications to reflectarray synthesis for radar cross-section reduction and control. Exploration of the radiation properties of array configurations based on other categories of aperiodic sequences continues.

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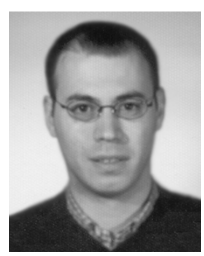

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