Rejection properties of stochastic-resonance-based detectors of weak harmonic signals

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In [V. Galdi *et al.*, Phys. Rev. E **57**, 6470 (1998)] a thorough characterization in terms of receiver operating characteristics of stochastic-resonance detectors of weak harmonic signals of known frequency in additive Gaussian noise was given. It was shown that strobed sign-counting based strategies can be used to achieve a nice trade-off between performance and cost, by comparison with noncoherent correlators. Here we discuss the more realistic case where besides the sought signal (whose frequency is assumed known) further unwanted spectrally nearby signals with comparable amplitude are present. Rejection properties are discussed in terms of suitably defined false-alarm and false-dismissal probabilities for various values of interfering signal(s) strength and spectral separation.

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I. BACKGROUND

Stochastic resonance (henceforth SR) is a peculiar phenomenon observed in a wide variety of physical systems [1] acted by a mixture of a time-harmonic signal and (white, Gaussian) noise, whereby the output spectral amplitude at the signal frequency shows a nonmonotonic dependence (i) on the noise strength at fixed signal frequency and amplitude, and (ii) on the signal frequency, at fixed signal and noise levels.

The possible use of SR in connection with weak signal detection experiments has been repeatedly suggested.

The only meaningful comparison between different detection strategies is to compare the related detection probabilities at the same level of false-alarm probabilities and available signal-to-noise ratios. In this connection, as demonstrated in Refs. [3,4], SR based detectors do not outperform matched filters. In Ref. [3] it was further shown that using SR as a preprocessor (signal-to-noise ratio enhancer) does not improve the performance of a matched-filter detector.

Nonetheless, SR detectors based on strobed sign-counting could be interesting as computationally cheap alternatives to noncoherent correlators, as discussed in Ref. [3] and summarized in the next section.

II. A STROBED SIGN COUNTING SR DETECTOR

The possibly simplest SR paradigm is the Langevin system [2]:

$$\dot{x} = -\frac{dV(x)}{dx} + A\sin(\omega_s t + \phi) + \epsilon n(t),$$

$$x(0) = x_0, \tag{1}$$

with quartic symmetric potential,

$$V(x) = -a\frac{x^2}{2} + b\frac{x^4}{4}, \quad a,b > 0, \tag{2}$$

where $n(t)$ is a stationary, zero mean, white Gaussian noise, with autocorrelation $E[n(t)n(t+\tau)] = \delta(\tau)$.

The probability density function (henceforth PDF) of $x(t)$ in (1), denoted as $p(x,t)$ is ruled by the Fokker–Planck equation,

$$\frac{\partial p(x,t)}{\partial t} = \frac{\partial}{\partial x}\left\{\left[\frac{dV(x)}{dx} - A\sin(\omega_s t + \phi)\right]p(x,t)\right\} + \frac{\epsilon^2}{2}\frac{\partial^2 p(x,t)}{\partial x^2},$$

$$p(x,0) = \delta(x - x_0). \tag{3}$$

The solution of (3), in the absence of a signal $(A=0)$ is an even function of x [3]. In the presence of a signal, even symmetry is broken, and the asymmetry is maximum at

$$t = t_k = \omega_s^{-1}\left(\frac{2k+1}{2}\pi - \psi\right), \quad k = 0,1,2,\ldots,2N-1, \tag{4}$$

where ψ is a (known [3]) phase-lag introduced by the SR processor.

Symmetry breaking of the output PDF is perhaps [5] the most natural signature of $A \neq 0$ in (1). In Ref. [3] we gave a thorough evaluation in terms of receiver (detection) operating characteristics (henceforth ROCs) of the possibly simplest nonparametric detector based on the above symmetry breaking, where from the output samples (4), one forms the time series

$$x_k = (-)^k x(t_k), \tag{5}$$

and compares

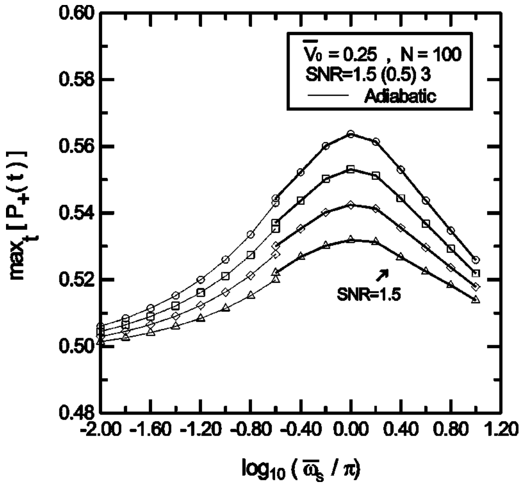

FIG. 1. Frequency response of SSC–SR detector for several values of SNR. Dashed curves refer to the adiabatic approximation (quoted from [3]).

$$N_+ = \sum_{k=0}^{2N-1} U(x_k), \tag{6}$$

where $U(\cdot)$ is Heaviside's step function, to a suitable threshold Γ. The above will be henceforth referred to as strobed sign-counting stochastic resonance detector (SSC–SRD).

The SSC–SRD performance is described by the following false-alarm and false-dismissal probabilities [3]:

$$\alpha = \mathrm{Prob}\{N_+ > \Gamma | \text{no signal}\} = I_{1/2}(\Gamma + 1, 2N - \Gamma), \tag{7}$$

$$\beta = \mathrm{Prob}\{N_+ \leqslant \Gamma | \text{signal}\} = 1 - I_{P_+}(\Gamma + 1, 2N - \Gamma), \tag{8}$$

where $P_+ = \mathrm{Prob}(x_k > 0)$ and $I_p(x,y)$ is the incomplete beta function. The related ROCs are typically worse by ≈ 3 dB as compared to those of the matched filter [3].

Generalization to the case where the initial phase of the sought signal is unknown is straightforward, by letting

$$N_+ = \max_{m \in (0,N_s)} \sum_{k=0}^{2N-1} U\left[(-)^k x\left(t_k + \frac{mT_s}{2N_s}\right)\right],$$
$$\text{where } T_s = 2\pi/\Omega_s. \tag{9}$$

The resulting unknown-initial-phase detector for $N_s \gtrsim 10$ has nearly the same performance as (6), which applies to the coherent (known initial phase) case [3], and is accordingly comparable to that of the noncoherent correlator (std. optimum benchmark detector for signals with unknown initial phase).

On the other hand, the SSC–SRD is computationally extremely cheap, requiring only binary and/or integer arithmetics, and thus quite appealing.

The obvious question is now related to the rejection properties of the above detector, namely to its ability to discriminate between spectrally nearby signals (sought and unwanted) with comparable amplitudes.

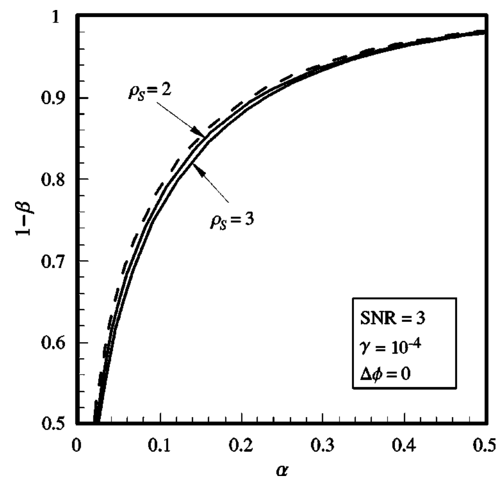

FIG. 2. ROCs of SSC–SR detector. SNR=3, $\gamma = 10^{-4}$, $\Delta\phi = 0$, $N = 100$. The $\rho_S = 0$ curve is shown dashed.

Note in this connection that the frequency response of the SSC–SRD depends very little on the stochastic resonance condition, as shown in Fig. 1 (see Ref. [3]), where the steady-state value of $\max_t[P_+(t)] = \max_t[\mathrm{Prob}(x(t) > 0)]$ is displayed for several values of the SNR as a function of $\overline{\omega_s} = \omega_s T_k$, where ω_s is the signal angular frequency and $T_k = \sqrt{2}\pi \exp(2\overline{V_0})/a$ is the Kramers time, $\overline{V_0} = a^2/(4b\epsilon^2)$ being the normalized potential-barrier height.

A numerical investigation of this issue will be the subject of the next section.

III. ROCs IN THE PRESENCE OF NEARBY SIGNALS

In order to evaluate the performance of the above described SSC–SRD in the presence of spectrally nearby signals with comparable amplitude, we introduce the following dimensionless parameters:

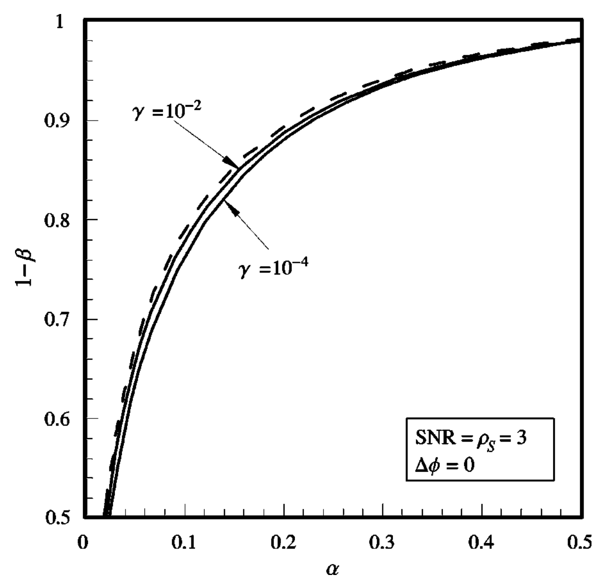

FIG. 3. ROCs of SSC–SR detector. SNR=3, $\rho_S = 3$, $\Delta\phi = 0$, $\gamma = 10^{-2}, 10^{-4}$, $N = 100$. The $\rho_S = 0$ curve is shown dashed.

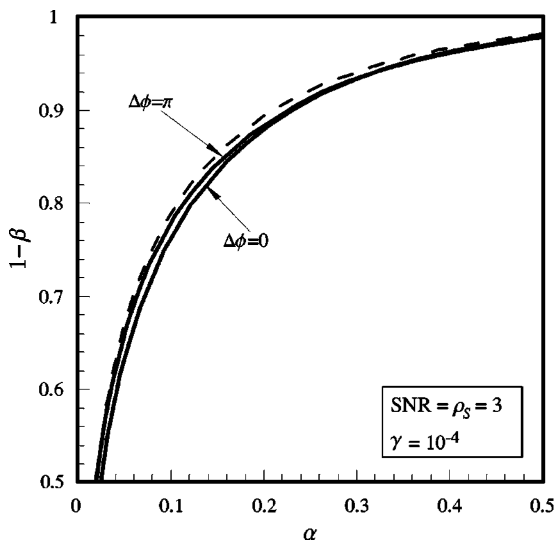

FIG. 4. ROCs of SSC–SR detector. SNR=3, $\rho_S=3$, $\gamma=10^{-4}$, $\Delta\phi=0,\pi$, $N=100$. The $\rho_S=0$ curve is shown dashed.

$$\rho_S = \frac{A_u}{A_w}, \quad \gamma = \left|\frac{\omega_w - \omega_u}{\omega_w}\right|, \quad \Delta\phi = \phi_w - \phi_u, \tag{10}$$

representing the unwanted-to-wanted (sought) signal-to-signal ratio (SSR), the (scaled) frequency difference, and the phase lag, where the suffixes w and u refer to the wanted and unwanted signal, respectively. More or less obviously, we also define the false-alarm (α) and false-dismissal (β) probabilities as follows:

$$\alpha = \mathrm{Prob}\{N_+ > \Gamma | s_w(t) = 0 \text{ and } s_u(t) \neq 0\}, \tag{11}$$

$$\beta = \mathrm{Prob}\{N_+ < \Gamma | s_w(t) \neq 0 \text{ and } s_u(t) \neq 0\}. \tag{12}$$

Representative numerical simulations [6] are accordingly summarized in Figs. 2–5 below.

Figure 2 describes a situation where SNR=3, and the unwanted signal frequency is pretty close to the sought one ($\gamma=10^{-4}$). Only two detection characteristics are displayed, corresponding to $\rho_S=2,3$ for the sake of readability, together with the limiting curve corresponding to the absence of the unwanted signal (dashed line). It is seen that the detector's performance is only slightly deteriorated due to the presence of the unwanted signal.

In Fig. 3 the SSR is fixed at ($\rho_S=3$), and two different (normalized) frequency separations $\gamma=10^{-2}$, 10^{-4} are considered. The limiting curve corresponding to the absence of the unwanted signal (dashed line) is also included. Again, the detector's performance is not appreciably spoiled.

In Fig. 4 the unwanted signal amplitude and spectral separation are fixed, $\rho_S=3$ and $\gamma=10^{-4}$, and two values of the phase lag are considered, $\Delta\phi=0$, π. Once more, the detector's performance is negligibly affected.

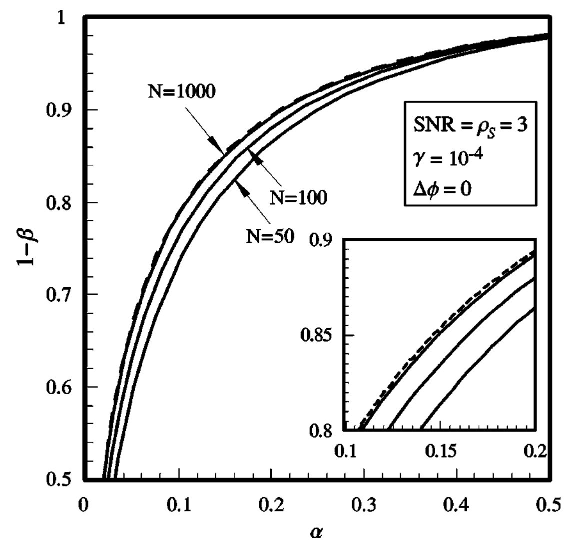

FIG. 5. ROCs of SSC–SR detector. SNR=3, $\rho_S=3$, $\gamma=10^{-4}$, $\Delta\phi=0$, $N=50,100,1000$. The $\rho_S=0$ curve is shown dashed. Close-up in the inset.

Finally, Fig. 5 shows the key role played by the number of strobed samples used in (9) at fixed $\rho_S=3$, $\gamma=10^{-4}$, $\Delta\phi=0$ on the SSC–SR detector's rejection performance. It is seen that the unwanted signal is rejected for $N>10^3$, for which the time series length corresponds to the spectral width required to separate the wanted and unwanted signals. This shows that the rejection properties are essentially related to the spectral filtering inherent to the strobing process.

IV. CONCLUSIONS AND RECOMMENDATIONS

Numerical simulations suggest that strobed-sign-counting stochastic-resonance-based detectors besides being computationally cheap, and nearly as much performing as the standard noncoherent correlator, do display nice rejection properties in the presence of spectrally nearby signals with comparable amplitudes.

Further study is under way to evaluate the potential of SSC–SR detectors in connection, e.g., with the search of weak nearby quasi-monochromatic signals in the context of the search of gravitational waves.

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[1] See, e.g., Ref. [2] for a survey of systems, signals and noises for which SR is predicted/observed.
[2] L. Gammaitoni *et al.*, Rev. Mod. Phys. **70**, 223 (1998).
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[6] F. Postiglione, thesis, University of Salerno, 1999.