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Derivation of higher-order impedance boundary conditions for stratified coatings composed of inhomogeneous-dielectric and homogeneous-bianisotropic layers

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Abstract. A spectral domain framework is presented for deriving exact electromagnetic impedance boundary conditions for isotropic, longitudinally inhomogeneous, dielectric coatings on a general (polarization-rotating) impedance plane. The derived expressions are shown to be well approximated over a reasonably wide range of parameters by means of rational functions of the spectral variables, from which higher-order approximate impedance boundary conditions are readily obtained by simple Fourier transformation. The proposed method is readily extended to multilayer coatings consisting of any combination of inhomogeneous dielectric layers and homogeneous, arbitrarily complex (e.g., bianisotropic, nonreciprocal) materials. Application to curved boundaries is also possible. A number of examples are included to validate the proposed approach and show its versatility and reliability.

1. Introduction

Modeling the effects of multilayer complex surface coatings is a key issue in a variety of problems including analysis of radar signatures [*Strifors and Gaunaurd*, 1998], shields [*Schulz et al*., 1988], absorbers [*Michielssen et al*., 1993], antennas [*Hakoropouros and Katehi*, 1991], and microwave devices [*Faché et al*., 1993].

In this connection, the use of recently synthesized complex materials (e.g., chiral [*Jaggard et al*., 1979; *Engheta and Jaggard*, 1988] and Ω-media [*Engheta and Saadoun*, 1992]) has received considerable attention [*Kluskens and Newman*, 1991; *Liu and Jaggard*, 1992; *Jaggard and Liu*, 1992; *Graglia et al*., 1992; *Norgren and He*, 1997].

Related design optimization procedures, both deterministic [*Fletcher*, 1980] and stochastic [*Davis*, 1987; *Weile and Michielssen*, 1997], usually require repeated evaluations of a merit function, i.e., repeated computation of the electromagnetic fields scattered by such coated bodies. Consequently, considerable effort has been devoted during the past decade to devise fast and accurate numerical techniques to perform this task. Most of these techniques fall into two categories.

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In the presence of (possibly piecewise) homogeneous coatings, one can formulate a combined field integral equation [*Huddleston et al*., 1986], which can be solved, for example, by the method of moments [*Harrington*, 1960]. Crucial to this approach is the (possibly analytical) knowledge of the Green's function of the coating. This turns out to be cumbersome, or even impossible, for complex materials. Moreover, the analysis of multilayer coatings leads to large matrix problems.

Inhomogeneous coatings can be handled by using finite element and related numerical methods [*Jin and Liepa*, 1988]. However, these methods are known to fail when the layer thickness is small compared with the wavelength, due to numerical instability.

In many applications, approximate impedance boundary conditions (IBCs), relating the tangential (or normal) components of the electric and magnetic fields at the interface between the coated object and free space through differential equations, whose coefficients are functions of the local properties of the coating, can be used effectively (see *Senior and Volakis* [1995] and *Hoppe and Rahmat-Samii* [1995] for a thorough review). The simplest conceivable versions, known as standard (or Leontovich) impedance boundary conditions (SIBC) [*Leontovich*, 1948] and tensor impedance boundary conditions (TIBC) [*Hoppe and Rahmat-Samii*, 1995], relate the tangential electric and magnetic fields by a constant factor (polarization-preserving coatings) or a tensor (polarization-rotating coatings), respectively. Both are derived by considering plane wave scattering at normal incidence, and their applicability is restricted to electrically thin and/or considerably lossy coatings.

Inclusion of (suitably high order) derivatives of the field components in the boundary conditions yields the so-called higher-order IBCs (HOIBCs) or generalized IBCs (GIBCs) [*Karp and Karal*, 1965; *Weinstein*, 1969; *Senior and Volakis*, 1989; *Rojas and Al-hekail*, 1989; *Idemen*, 1990; *Cicchetti*, 1996], whereby more general or thicker coatings can be modeled. (As usual in IBC literature, the order of an IBC is intended as the highest order of field derivatives involved.) HOIBCs/GIBCs have been successfully applied to a variety of scattering problems involving homogeneous [*Senior and Volakis*, 1987; *Volakis and Senior*, 1989] and inhomogeneous [*Ammari and He*, 1997; *Ammari and He*, 1998] dielectric layers, dielectric-filled grooves [*Barkeshli and Volakis*, 1990], and multilayer coatings [*Volakis and Syed*, 1990; *Ricoy and Volakis*, 1990; *Tretyakov*, 1998].

A spectral domain approach for systematically deriving HOIBCs for planar (as well as curved) coatings consisting of piecewise homogeneous materials with arbitrary (linear, bianisotropic) constitutive relations has been introduced by *Hoppe and Rahmat-Samii* [1992] and is thoroughly illustrated by *Hoppe and Rahmat-Samii* [1995]. As shown by the above authors, exact spectral domain IBCs for these kinds of coatings can be usually obtained in analytical (though complicate) form and can be well approximated by suitable low-order rational functions of the spectral variables, which can be readily Fourier-transformed to obtain the sought spatial domain HOIBCs. This approach turns out to be very attractive and particularly well suited for computer-aided design (CAD) applications, on account of its generality, wide applicability, and accuracy.

One of the main restrictions of the aforementioned approach is related to the impossibility of handling inhomogeneous (in the direction normal to the boundary) coatings. This restriction might be limiting, for example, for broadband optimization absorbers, where tapered layers could considerably improve the performance.

In a recent paper [*Galdi and Pinto*, 1999a] we proposed an efficient strategy to overcome this limitation, for inhomogeneous isotropic dielectric coatings laid on polarization-preserving impedance planes. In the present paper we go one step further, by considering multisegmented coatings consisting of any combination of inhomogeneous dielectric layers and homogeneous bianisotropic materials. To this end, we solve the canonical problem of electromagnetic scattering by an infinite general (polarization-rotating) impedance plane coated by an isotropic, inhomogeneous (possibly lossy) dielectric layer with an arbitrary permittivity profile. A power series expansion technique is exploited to solve the resulting (coupled) Stourm-Liouville problems ruling the fields in the dielectric layer for the two possible polarizations. The exact spectral domain IBCs thus obtained are then Fourier-transformed in the spatial domain by using suitable rational approximations for the spectral impedance components, as proposed by *Hoppe and Rahmat-Samii* [1995].

The remainder of the paper is organized as follows. In section 2 the analytical framework is introduced: HOIBCs for arbitrary bianisotropic layers are briefly reviewed, and the derivation of HOIBCs for an arbitrary isotropic, inhomogeneous dielectric coating on an impedance plane is outlined. In section 3 a number of numerical examples are presented and discussed, in order to validate the proposed method and show its main features. Conclusions follow in section 4. A body of technical details are collected in the appendices. An implicit exp $(i\omega t)$ time-harmonic dependence is assumed and suppressed throughout the paper.

2. Derivation of HOIBCs

2.1. Exact IBCs for Homogeneous Bianisotropic Layers

We first present a generalized derivation of the exact IBCs for a homogeneous bianisotropic coating first introduced by *Hoppe and Rahmat-Samii* [1995]. The pertinent geometry is illustrated in Figure 1a. We consider an impedance plane at $z = 0$ coated with a homogeneous layer of bianisotropic material, described by the (general) constitutive relations [*Kong*, 1975]

$$\mathbf{D} = \varepsilon_0(\bar{\varepsilon}_r \cdot \mathbf{E} + \eta_0 \bar{\xi} \cdot \mathbf{H})$$
$$\mathbf{B} = \mu_0(\bar{\mu}_r \cdot \mathbf{H} + \eta_0^{-1} \bar{\eta} \cdot \mathbf{E}), \tag{1}$$

where ε_0 and μ_0 are the free-space electric permittivity and magnetic permeability, respectively, η_0 is the free-space impedance, and $\bar{\varepsilon}_r$, $\bar{\mu}_r$, $\bar{\xi}$, $\bar{\eta}$ are dimensionless second-rank tensors. In the assumed Cartesian coordinate system, all vectors are repre-

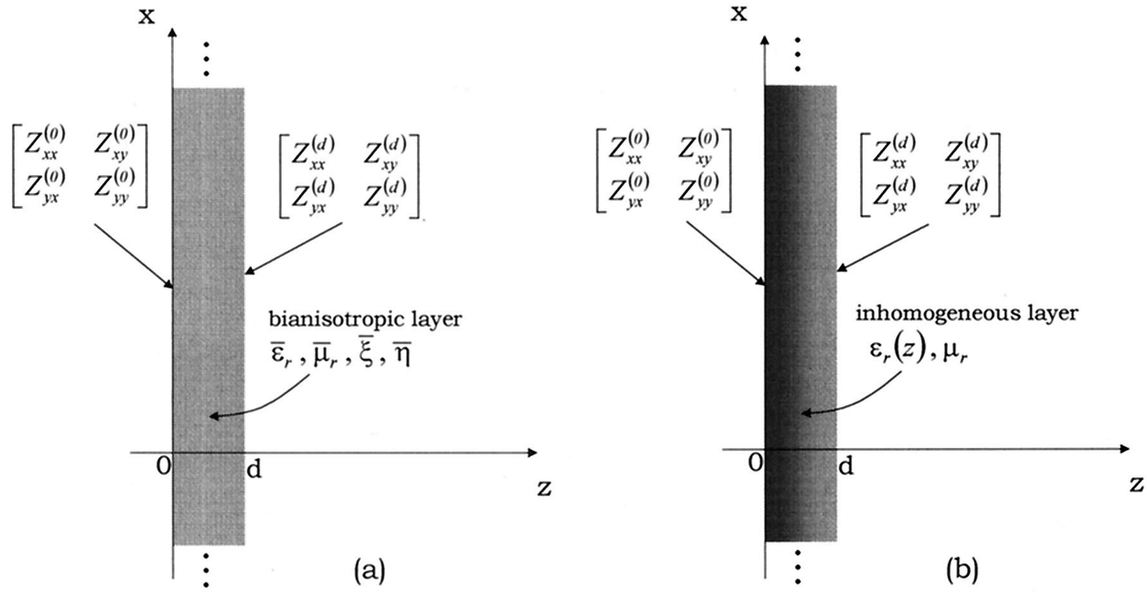


Figure 1. Geometry of canonical scattering problems.

sented as 3 × 1 (column) matrices, whereas all tensors are written as 3 × 3 matrices.

In the following, we shall use an impedance-matrix formulation in order to cope with the notation given by *Hoppe and Rahmat-Samii* [1995]. However, it could be worth pointing out that alternative formulations based, for example, on the (generalized) immittance approach [*Itoh*, 1980] or the generalized propagation matrix [*He and Takenaka*, 1998], well suited to handle layered structures, could be also fruitfully employed.

We face this problem in the spectral domain, i.e., by studying plane wave propagation in the layer. In the following, we shall focus our attention on rotationally symmetric coatings, whose IBCs are invariant under an arbitrary rotation around the z axis. This symmetry arises in many situations of practical interest (e.g., chiral layers, normally biased ferrite layers, normally uniaxial crystals), and it can be shown [*Hoppe and Rahmat-Samii*, 1995] that under this assumption solving the related one-dimensional problem ($\partial/\partial y = 0$) is sufficient to derive the most general IBCs.

Accordingly, the total field inside the coating can be expressed as a linear combination of four waves with unknown coefficients [*Hoppe and Rahmat-Samii*, 1995]:

$$\mathbf{E}(k_x, z) = \sum_{j=1}^{4} c_j \mathbf{e}_t^{(j)}(k_x) \exp\left[-i(k_x x + k_z^{(j)} z)\right]$$
$$\mathbf{H}(k_x, z) = \sum_{j=1}^{4} c_j \mathbf{h}_t^{(j)}(k_x) \exp\left[-i(k_x x + k_z^{(j)} z)\right], \tag{2}$$

where k_x is the transverse wavenumber (spectral variable). In the following, the $\exp(-ik_x x)$ dependence will be omitted for notational simplicity. The vector functions $\mathbf{e}_t^{(j)}(k_x)$, $\mathbf{h}_t^{(j)}(k_x)$ are the transverse parts of the characteristic waves, i.e., the source-free solutions of Maxwell's curl equations, namely [*Graglia et al.*, 1991],

$$[(\bar{\kappa} + k_0 \bar{\xi}) \cdot \bar{\mu}_r^{-1} \cdot (\bar{\kappa} - k_0 \bar{\eta}) + k_0^2 \bar{\varepsilon}_r] \cdot \mathbf{e} = 0, \tag{3}$$

$$\mathbf{h} = \frac{1}{k_0 \eta_0} \bar{\mu}_r^{-1} \cdot (\bar{\kappa} - k_0 \bar{\eta}) \cdot \mathbf{e}, \tag{4}$$

where $k_0 = 2\pi/\lambda_0$ is the free-space wavenumber and

$$\bar{\kappa} = \begin{bmatrix} 0 & -k_z & 0 \\ k_z & 0 & -k_x \\ 0 & k_x & 0 \end{bmatrix}. \tag{5}$$

The related propagation constants, $k_z^{(j)}$, are solutions of the (fourth-degree, algebraic) dispersion equation [*Graglia et al.*, 1991]:

$$\det\left[(\bar{\kappa} + k_0\bar{\xi}) \cdot \bar{\mu}_r^{-1} \cdot (\bar{\kappa} - k_0\bar{\eta}) + k_0^2\bar{\varepsilon}_r\right] = 0. \tag{6}$$

As already stated, the ground plane ($z = 0$) is described by a general tensor IBC:

$$\begin{bmatrix} E_x(k_x, 0) \\ E_y(k_x, 0) \end{bmatrix} = [Z^{(0)}(k_x)] \cdot \begin{bmatrix} H_x(k_x, 0) \\ H_y(k_x, 0) \end{bmatrix} = \begin{bmatrix} Z_{xx}^{(0)}(k_x) & Z_{xy}^{(0)}(k_x) \\ Z_{yx}^{(0)}(k_x) & Z_{yy}^{(0)}(k_x) \end{bmatrix} \cdot \begin{bmatrix} H_x(k_x, 0) \\ H_y(k_x, 0) \end{bmatrix}. \tag{7}$$

(We recall that [*Hoppe and Rahmat-Samii*, 1995] for reciprocal boundaries, Re $[Z_{xy}^{(0)}]$ = Re $[Z_{yx}^{(0)}] = 0$, $Z_{xx}^{(0)} = Z_{yy}^{(0)*}$, whereas for lossless boundaries $Z_{xx}^{(0)} = -Z_{yy}^{(0)}$.) Equations (7) can model the presence of a (possibly lossy, corrugated) metallic plane or represent the effects of further layers, so that extension to multilayer coatings is almost immediate and amounts to iterating the procedure below.

By generalizing the procedure of *Hoppe and Rahmat-Samii* [1995] (see Appendix A), the exact IBCs at the interface $z = d$ can be readily obtained, namely,

$$[Z^{(d)}(k_x)] := \begin{bmatrix} Z_{xx}^{(d)}(k_x) & Z_{xy}^{(d)}(k_x) \\ Z_{yx}^{(d)}(k_x) & Z_{yy}^{(d)}(k_x) \end{bmatrix} = ([E_{12}(k_x, d)] + [E_{34}(k_x, d)] \cdot [M^{(0)}(k_x)]) \cdot ([H_{12}(k_x, d)] + [H_{34}(k_x, d)] \cdot [M^{(0)}(k_x)])^{-1}, \tag{8}$$

where

$$[M^{(0)}(k_x)] = ([Z^{(0)}(k_x)] \cdot [H_{34}(k_x, 0)] - [E_{34}(k_x, 0)])^{-1} \cdot ([E_{12}(k_x, 0)] - [Z^{(0)}(k_x)] \cdot [H_{12}(k_x, 0)]), \tag{9}$$

$$[E_{pq}(k_x, z)] = \begin{bmatrix} \hat{i}_x \cdot \mathbf{e}_t^{(p)}(k_x) \exp(-ik_z^{(p)}z) & \hat{i}_x \cdot \mathbf{e}_t^{(q)}(k_x) \exp(-ik_z^{(q)}z) \\ \hat{i}_y \cdot \mathbf{e}_t^{(p)}(k_x) \exp(-ik_z^{(p)}z) & \hat{i}_y \cdot \mathbf{e}_t^{(q)}(k_x) \exp(-ik_z^{(q)}z) \end{bmatrix}, \tag{10}$$

$$[H_{pq}(k_x, z)] = \begin{bmatrix} \hat{i}_x \cdot \mathbf{h}_t^{(p)}(k_x) \exp(-ik_z^{(p)}z) & \hat{i}_x \cdot \mathbf{h}_t^{(q)}(k_x) \exp(-ik_z^{(q)}z) \\ \hat{i}_y \cdot \mathbf{h}_t^{(p)}(k_x) \exp(-ik_z^{(p)}z) & \hat{i}_y \cdot \mathbf{h}_t^{(q)}(k_x) \exp(-ik_z^{(q)}z) \end{bmatrix}, \tag{11}$$

where $\hat{i}_x$, $\hat{i}_y$ are the x- and y-directed unit vectors, respectively.

2.2. Exact IBCs for Inhomogeneous Dielectric Layers

Let us next consider the electromagnetic scattering by an infinite impedance plane at $z = 0$ coated by an arbitrary isotropic, longitudinally inhomogeneous (possibly lossy) dielectric layer, as depicted in Figure 1b. The relative electric permittivity profile is written [*Snyder and Love*, 1983]

$$\varepsilon_r(z) = \varepsilon_r^{(0)}[1 - 2\Delta f(z)], \qquad \Delta = \frac{\varepsilon_r^{(0)} - \varepsilon_r^{(d)}}{2\varepsilon_r^{(0)}}. \tag{12}$$

The function $f(z)$ describes the z inhomogeneity and is assumed to be analytic in $0 \le z \le d$, with $f(0) = 0$, $f(d) = 1$, where d is the layer thickness. Moreover, without loss in generality, we shall assume $f(z)$ to be monotonic and $\varepsilon_r^{(0)} > \varepsilon_r^{(d)}$. Again, the one-dimensional problem is considered, and the ground plane is described by the tensor IBC (equation (7)).

As is well known [*Snyder and Love*, 1983], the dielectric slab under analysis supports two different polarizations, namely, transverse electric and transverse magnetic. Accordingly, the z dependence of the transverse field components inside the coating is governed by the following Sturm-Liouville problems [*Snyder and Love*, 1983]:

$$\left\{\frac{\partial^2}{\partial z^2} + k_z^2 - [\varepsilon_r^{(0)} - \varepsilon_r^{(d)}]\mu_r k_0^2 f(z)\right\} E_y(k_x, z) = 0, \tag{13}$$

$$\left\{\frac{\partial^2}{\partial z^2} - \frac{1}{\varepsilon_r(z)}\frac{d\varepsilon_r(z)}{dz}\frac{\partial}{\partial z} + k_z^2 - [\varepsilon_r^{(0)} - \varepsilon_r^{(d)}]\mu_r k_0^2 f(z)\right\} H_y(k_x, z) = 0, \tag{14}$$

under boundary conditions (7), and

$$H_x(k_x, z) = -\frac{i}{k_0\eta_0\mu_r}\frac{\partial E_y(k_x, z)}{\partial z}, \tag{15}$$

$$E_x(k_x, z) = \frac{i\eta_0}{k_0\varepsilon_r(z)}\frac{\partial H_y(k_x, z)}{\partial z}. \tag{16}$$

(Note that equations (13) and (14) are actually coupled by the IBCs (7) at the ground plane, in the general (polarization-rotating) case $Z_{xx}^{(0)}$, $Z_{yy}^{(0)} \neq 0$.)

In (13)–(16), μ_r is the (constant) relative magnetic permeability of the layer, and $k_z^2 = (\varepsilon_r^{(0)}\mu_r k_0^2 - k_x^2)$.

Since $f(z)$ is analytic and $\varepsilon_r(z)$ does not vanish in $[0, d]$, equations (13) and (14) admit a convergent Taylor series solution in $0 \le z \le d$ [*Bender and Orszag*, 1978]. By enforcing the McLaurin expansions

$$f(z) = \sum_{n=1}^{\infty} f_n (k_0 z)^n, \quad \frac{1}{\varepsilon_r(z)} \frac{d\varepsilon_r(z)}{dz} = k_0 \sum_{n=0}^{\infty} g_n (k_0 z)^n, \tag{17}$$

$$E_y(k_x, z) = \psi_e(k_x, z, a_0, a_1) := \sum_{n=0}^{\infty} a_n (k_0 z)^n, \tag{18}$$

$$H_y(k_x, z) = \psi_h(k_x, z, b_0, b_1) := \sum_{n=0}^{\infty} b_n (k_0 z)^n, \tag{19}$$

into the Sturm-Liouville problems (13) and (14), differentiating repeatedly the resulting identity, and setting $z = 0$, so as to equate to zero all coefficients of the resulting power series in $k_0 z$, one readily gets a recursion relation for the unknown coefficients a_n, b_n [*Bender and Orszag*, 1978]:

$$a_n = \frac{(\varepsilon_r^{(0)} - \varepsilon_r^{(d)})\mu_r \sum_{j=1}^{n-2} f_j a_{n-j-2} - \left(\varepsilon_r^{(0)}\mu_r - \frac{k_x^2}{k_0^2}\right) a_{n-2}}{n(n-1)}, \quad n \ge 2, \tag{20}$$

$$b_n = \frac{(\varepsilon_r^{(0)} - \varepsilon_r^{(d)})\mu_r \sum_{j=1}^{n-2} f_j b_{n-j-2}}{n(n-1)} + \frac{\sum_{j=0}^{n-1} (n-j-1) g_j b_{n-j-1} - \left(\varepsilon_r^{(0)}\mu_r - \frac{k_x^2}{k_0^2}\right) b_{n-2}}{n(n-1)}, \quad n \ge 2. \tag{21}$$

The general solutions of (13) and (14) can be written as linear combinations of two independent solutions, for example,

$$E_y^{(1)}(k_x, z) := \psi_e(k_x, z, 0, 1), \qquad E_y^{(2)}(k_x, z) := \psi_e(k_x, z, 1, 0), \tag{22}$$

$$H_y^{(1)}(k_x, z) := \psi_h(k_x, z, 0, 1), \qquad H_y^{(2)}(k_x, z) := \psi_h(k_x, z, 1, 0). \tag{23}$$

One gets

$$\begin{bmatrix} E_x(k_x, z) \\ E_y(k_x, z) \end{bmatrix} = [E_1(k_x, z)] \cdot \begin{bmatrix} A \\ C \end{bmatrix} + [E_2(k_x, z)] \cdot \begin{bmatrix} B \\ D \end{bmatrix}, \tag{24}$$

$$\begin{bmatrix} H_x(k_x, z) \\ H_y(k_x, z) \end{bmatrix} = [H_1(k_x, z)] \cdot \begin{bmatrix} A \\ C \end{bmatrix} + [H_2(k_x, z)] \cdot \begin{bmatrix} B \\ D \end{bmatrix}, \tag{25}$$

where A, B, C, and D are arbitrary constants and

$$[E_j(k_x, z)] = \begin{bmatrix} \frac{i\eta_0}{k_0 \varepsilon_r(z)} \frac{\partial H_y^{(j)}(k_x, z)}{\partial z} & 0 \\ 0 & E_y^{(j)}(k_x, z) \end{bmatrix}, \quad j = 1, 2, \tag{26}$$

$$[H_j(k_x, z)] = \begin{bmatrix} 0 & -\frac{i}{k_0 \eta_0 \mu_r} \frac{\partial E_y^{(j)}(k_x, z)}{\partial z} \\ H_y^{(j)}(k_x, z) & 0 \end{bmatrix}, \quad j = 1, 2. \tag{27}$$

By enforcing the IBCs (7) at the ground plane, the coefficients A and C can be determined in terms of B and D, namely,

$$\begin{bmatrix} A \\ C \end{bmatrix} = [Q^{(0)}(k_x)] \cdot \begin{bmatrix} B \\ D \end{bmatrix}, \tag{28}$$

where

$$[Q^{(0)}(k_x)] = ([Z^{(0)}(k_x)] \cdot [H_1(k_x, 0)] - [E_1(k_x, 0)])^{-1} \cdot ([E_2(k_x, 0)] - [Z^{(0)}(k_x)] \cdot [H_2(k_x, 0)]). \tag{29}$$

The sought IBCs at the interface $z = d$ can be finally obtained by eliminating the coefficients B and D in (24) and (25), namely,

$$[Z^{(d)}(k_x)] = ([E_1(k_x, d)] \cdot [Q^{(0)}(k_x)] + [E_2(k_x, d)]) \cdot ([H_1(k_x, d)] \cdot [Q^{(0)}(k_x)] + [H_2(k_x, d)])^{-1}. \tag{30}$$

Obviously, in the presence of a polarization-preserving ground plane for which $Z_{xx}^{(0)} = Z_{yy}^{(0)} = 0$, one recovers the simpler result reported by *Galdi and Pinto* [1999a].

2.3. Polynomial Approximation

By suitably iterating the above procedures, exact IBCs can be systematically obtained for multilayered coatings consisting of an arbitrary number of stacked homogeneous bianisotropic and inhomogeneous dielectric layers. These IBCs could be Fourier-transformed in order to obtain the exact spatial domain boundary conditions. Unfortunately, this task can be rarely accomplished in a useful form in view of their complicated nature. One is therefore led to introduce suitable approximations. In this connection, *Hoppe and Rahmat-Samii* [1995] proposed the use of polynomial approximations (in the spectral variable k_x), which can be readily Fourier-transformed ($k_x \rightarrow id/dx$). Following this approach, the exact spectral domain IBCs are approximated by

$$\begin{bmatrix} P_1(k_x) & P_2(k_x) \\ P_3(k_x) & P_4(k_x) \end{bmatrix} \begin{bmatrix} E_x(k_x, d) \\ E_y(k_x, d) \end{bmatrix} \approx \begin{bmatrix} P_5(k_x) & P_6(k_x) \\ P_7(k_x) & P_8(k_x) \end{bmatrix} \begin{bmatrix} H_x(k_x, d) \\ H_y(k_x, d) \end{bmatrix} \tag{31}$$

or, equivalently,

$$\begin{bmatrix} P_1(k_x) & P_2(k_x) \\ P_3(k_x) & P_4(k_x) \end{bmatrix} \begin{bmatrix} Z_{xx}^{(d)}(k_x) & Z_{xy}^{(d)}(k_x) \\ Z_{yx}^{(d)}(k_x) & Z_{yy}^{(d)}(k_x) \end{bmatrix} \approx \begin{bmatrix} P_5(k_x) & P_6(k_x) \\ P_7(k_x) & P_8(k_x) \end{bmatrix}, \tag{32}$$

where $P_1(k_x) - P_8(k_x)$ are polynomials in k_x. Accordingly, the approximate spectral domain IBCs can be written as

$$\begin{bmatrix} \tilde{Z}_{xx}^{(d)}(k_x) & \tilde{Z}_{xy}^{(d)}(k_x) \\ \tilde{Z}_{yx}^{(d)}(k_x) & \tilde{Z}_{yy}^{(d)}(k_x) \end{bmatrix} = \begin{bmatrix} P_1(k_x) & P_2(k_x) \\ P_3(k_x) & P_4(k_x) \end{bmatrix}^{-1} \cdot \begin{bmatrix} P_5(k_x) & P_6(k_x) \\ P_7(k_x) & P_8(k_x) \end{bmatrix}, \tag{33}$$

where each impedance term is approximated by a rational function of k_x. In the following, we shall focus our attention on the second-order case, i.e.,

$$P_n(k_x) = c_0^{(n)} + c_1^{(n)} k_x + c_2^{(n)} k_x^2, \quad n = 1, \cdots, 8. \tag{34}$$

Because of the assumed rotational invariance of the IBCs, the above polynomials take a simpler form [*Hoppe and Rahmat-Samii*, 1995]:

$$P_1(k_x) = 1 + c_2^{(1)} k_x^2, \quad P_2(k_x) = c_2^{(2)} k_x^2,$$
$$P_3(k_x) = c_2^{(3)} k_x^2, \quad P_4(k_x) = 1 + c_2^{(4)} k_x^2, \tag{35}$$

$$P_5(k_x) = c_0^{(5)} + c_2^{(5)} k_x^2, \quad P_6(k_x) = c_0^{(6)} + c_2^{(6)} k_x^2, \tag{36}$$

$$P_7(k_x) = -c_0^{(6)} + c_2^{(7)} k_x^2, \quad P_8(k_x) = c_0^{(5)} + c_2^{(8)} k_x^2. \tag{37}$$

The ten unknown coefficients in (35)–(37) can be computed by matching the exact impedances, given by (8) or (30), and the approximate ones (equation (32)), at normal incidence ($k_x = 0$) and at two distinct values of k_x (e.g., $k_x^{(1)} = k_0/2$ and $k_x^{(2)} = k_0$, so as to span the whole visible range), as suggested by *Hoppe and Rahmat-Samii* [1995]. (Different choices can be exploited to span the surface-wave range, too [*Hoppe and Rahmat-Samii*, 1995; *Galdi and Pinto*, 1999a].) The complete solution is reported in Appendix B.

2.4. Spatial Domain HOIBCs

Once the coefficients are computed, the second-order spatial domain IBCs can be written as

$$\left(1 - c_2^{(1)} \frac{d^2}{dx^2}\right) E_x(x) - c_2^{(2)} \frac{d^2 E_y(x)}{dx^2} = \left(c_0^{(5)} - c_2^{(5)} \frac{d^2}{dx^2}\right) H_x(x) + \left(c_0^{(6)} - c_2^{(6)} \frac{d^2}{dx^2}\right) H_y(x) \tag{38}$$

$$-c_2^{(3)} \frac{d^2 E_x(x)}{dx^2} + \left(1 - c_2^{(4)} \frac{d^2}{dx^2}\right) E_y(x) = -\left(c_0^{(6)} + c_2^{(7)} \frac{d^2}{dx^2}\right) H_x(x) + \left(c_0^{(5)} - c_2^{(8)} \frac{d^2}{dx^2}\right) H_y(x). \tag{39}$$

As already anticipated, owing to rotational invariance, two-dimensional (2-D) HOIBCs follow by simple symmetry considerations [*Hoppe and Rahmat-Samii*, 1995]. One gets

$$\left[1 - c_2^{(1)} \frac{\partial^2}{\partial x^2} - c_2^{(4)} \frac{\partial^2}{\partial y^2} + (c_2^{(3)} + c_2^{(2)}) \frac{\partial^2}{\partial x \partial y}\right] E_x(x, y) + \left[-c_2^{(2)} \frac{\partial^2}{\partial x^2} + c_2^{(3)} \frac{\partial^2}{\partial y^2} - (c_2^{(1)} - c_2^{(4)}) \frac{\partial^2}{\partial x \partial y}\right] E_y(x, y)$$
$$= \left[c_0^{(5)} - c_2^{(5)} \frac{\partial^2}{\partial x^2} - c_2^{(8)} \frac{\partial^2}{\partial y^2} + (c_2^{(6)} + c_2^{(7)}) \frac{\partial^2}{\partial x \partial y}\right] H_x(x, y) + \left[c_0^{(6)} - c_2^{(6)} \frac{\partial^2}{\partial x^2} + c_2^{(7)} \frac{\partial^2}{\partial y^2} - (c_2^{(5)} - c_2^{(8)}) \frac{\partial^2}{\partial x \partial y}\right] H_y(x, y) \tag{40}$$

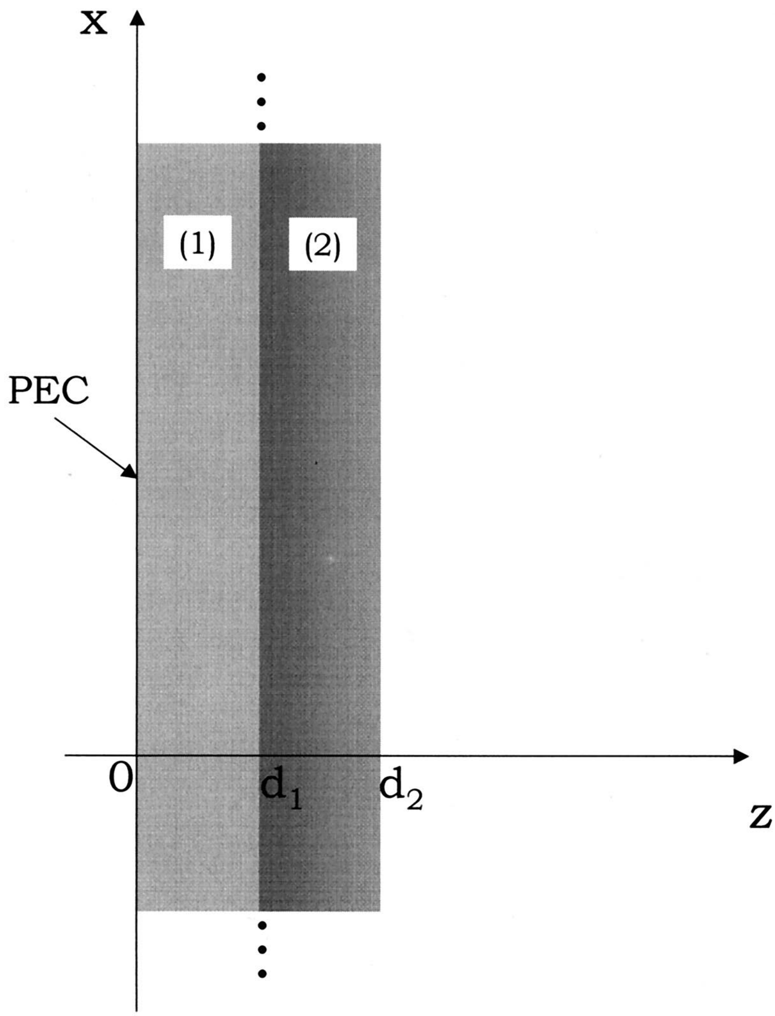


Figure 2. Two-layer coating. For layer 1 (chiral), $\varepsilon_r = 4$, $\mu_r = 1.5$, $\gamma_c = 0.005\ \Omega^{-1}$, and $l_1 = d_1 = 0.05\lambda_0$. For layer 2 (linear-profile dielectric), $\varepsilon_r^{(d_1)} = 4$, $\varepsilon_r^{(d_2)} = 1$, $\mu_r = 1$, and $l_2 = d_2 - d_1 = 0.05\lambda_0$.

$$\left[-c_2^{(3)}\frac{\partial^2}{\partial x^2} + c_2^{(2)}\frac{\partial^2}{\partial y^2} - (c_2^{(1)} - c_2^{(4)})\frac{\partial^2}{\partial x \partial y}\right]E_x(x, y)$$
$$+\left[1 - c_2^{(4)}\frac{\partial^2}{\partial x^2} - c_2^{(1)}\frac{\partial^2}{\partial y^2} - (c_2^{(3)} + c_2^{(2)})\frac{\partial^2}{\partial x \partial y}\right]E_y(x, y)$$
$$= \left[-c_0^{(6)} - c_2^{(7)}\frac{\partial^2}{\partial x^2} + c_2^{(6)}\frac{\partial^2}{\partial y^2} - (c_2^{(5)} - c_2^{(8)})\frac{\partial^2}{\partial x \partial y}\right]H_x(x, y)$$
$$+\left[c_0^{(5)} - c_2^{(8)}\frac{\partial^2}{\partial x^2} - c_2^{(5)}\frac{\partial^2}{\partial y^2} - (c_2^{(6)} + c_2^{(7)})\frac{\partial^2}{\partial x \partial y}\right]H_y(x, y). \quad (41)$$

We note that (smooth) nonplanar boundaries and (slowly varying) transverse inhomogeneities can be handled by using position-dependent coefficients. In the presence of abrupt variations or discontinuities of the coating parameters, however, additional requirements (e.g., Meixner-like conditions) need to be enforced in order to guarantee the uniqueness of the solution, as discussed, for example, by *Senior* [1992]. Application to curved coatings is possible by consistently using a locally planar approximation. In fact, it has been shown [see, e.g., *Hoppe and Rahmat-Samii*, 1995] that quite accurate results are generally ob-

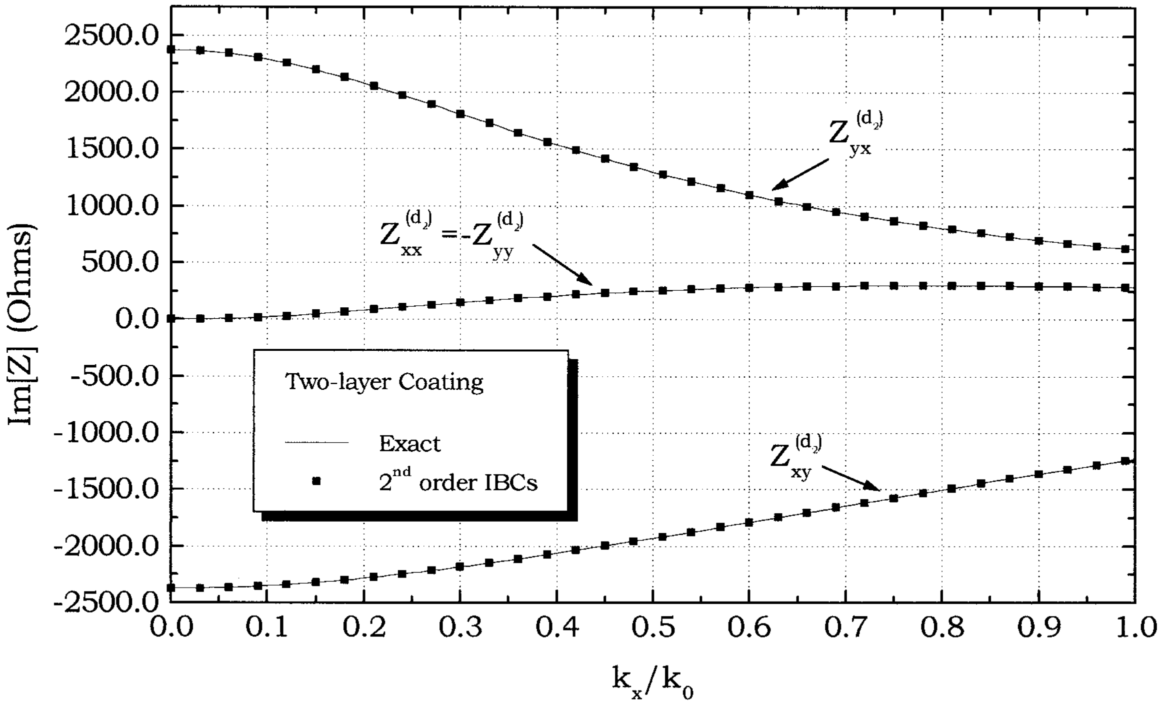


Figure 3. Impedance boundary conditions (IBCs) for the two-layer coating of Figure 2.

tained provided the (local) curvature radius is comparable to (or larger than) a wavelength, so that in our application examples we restrict our attention to canonical planar layered configurations.

3. Numerical Results

An important preliminary issue for applying and testing the proposed HOIBCs involves a suitable truncation of the summations ($E_y^{(1,2)}$, $H_y^{(1,2)}$ and their derivatives) appearing in the exact IBCs (equation (30)), whose evaluation is required for (point-matching) coefficient computation and, obviously, as a benchmark solution to evaluate the accuracy of the obtained HOIBCs. In all simulations presented in this paper we included 100 terms, which were found to guarantee a five-digit accuracy.

As a first application example we consider a two-layer coating consisting of a homogeneous chiral layer and a linear-profile inhomogeneous dielectric layer on a perfectly conducting ground plane, as depicted in Figure 2. The whole structure is assumed lossless in order to get a more significant test. We recall that a chiral medium is described by the following constitutive relations [*Jaggard et al.*, 1979; *Engheta and Jaggard*, 1988]:

$$\mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E} - i\gamma_c \mathbf{B}, \qquad \mathbf{H} = \frac{1}{\mu_0 \mu_r}\mathbf{B} - i\gamma_c \mathbf{E}, \tag{42}$$

where γ_c is the chiral admittance. The needed (transverse) characteristic waves and related propagation constants are reported in Appendix C. The linear-profile dielectric layer is described by

$$\varepsilon_r(z) = \varepsilon_r^{(d_1)}\left[1 - \frac{(z-d_1)(\varepsilon_r^{(d_1)} - \varepsilon_r^{(d_2)})}{\varepsilon_r^{(d_1)}(d_2 - d_1)}\right], \qquad d_1 < z < d_2. \tag{43}$$

Accordingly, (8)–(10) can be used to compute the exact IBCs at the interface between the chiral layer and the dielectric one ($z = d_1$). The exact IBCs at the interface $z = d_2$ are then obtained by using (30) and are finally approximated as rational functions, as illustrated in section 2. In Figure 3 the exact impedance terms and their second-order approximations (purely imaginary, as required for reciprocal, lossless coatings) are displayed as functions of the scaled wavenumber. A uniformly high accuracy is observed

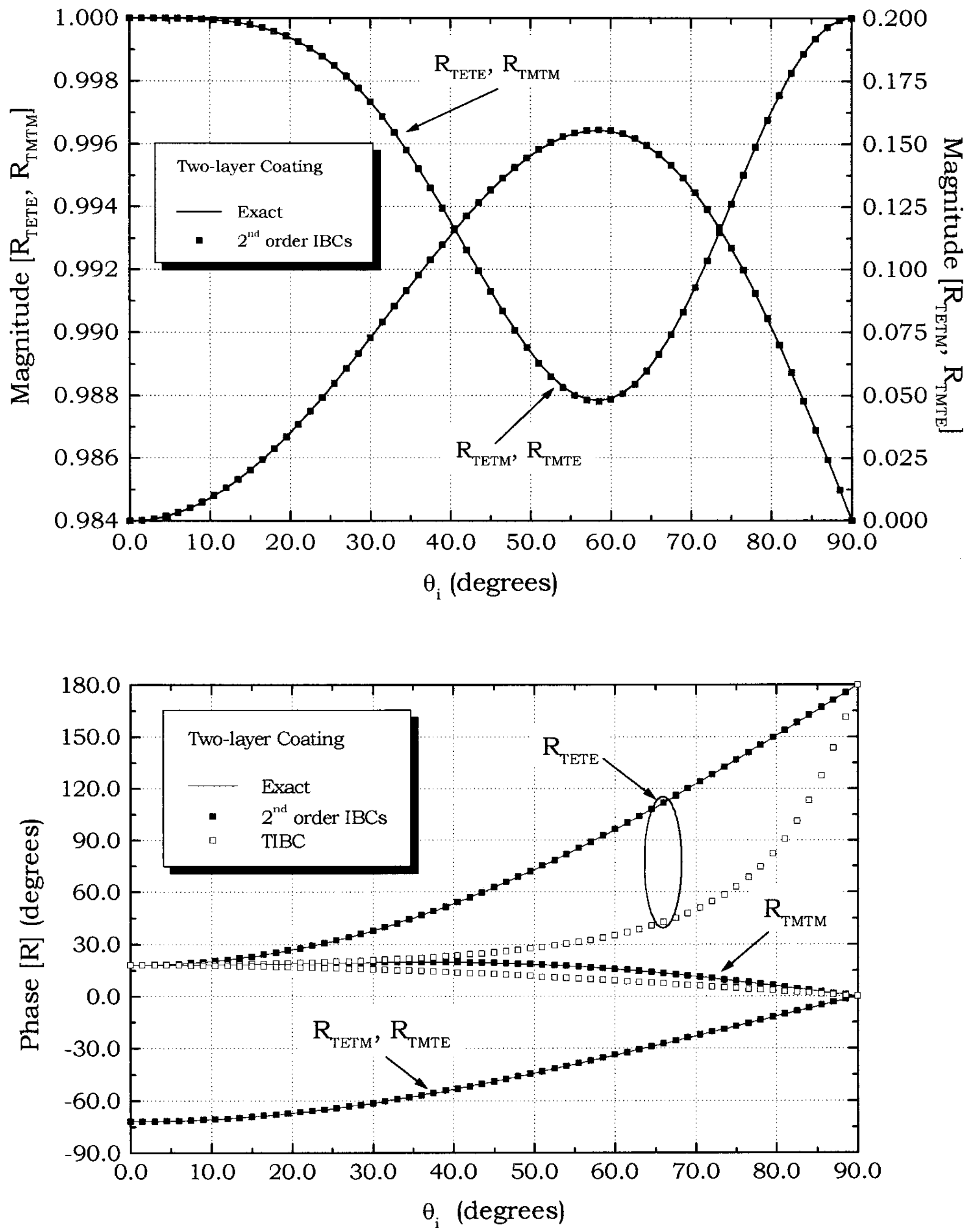


Figure 4. Plane wave reflection coefficients for the two-layer coating of Figure 2.

over the whole visible range. As a further accuracy test, we consider the one-dimensional scattering of a plane wave impinging from free space on the coated plane under analysis, as suggested by *Senior and Volakis* [1989] and *Hoppe and Rahmat-Samii* [1995], and compute the reflection coefficients:

$$\begin{bmatrix} R_{\mathrm{TETE}} & R_{\mathrm{TETM}} \\ R_{\mathrm{TMTE}} & R_{\mathrm{TMTM}} \end{bmatrix} = -([M_E(k_x)] + [Z^{(d_2)}(k_x)] \cdot [M_H(k_x)])^{-1} \cdot ([M_E(k_x)] - [Z^{(d_2)}(k_x)] \cdot [M_H(k_x)]), \tag{44}$$

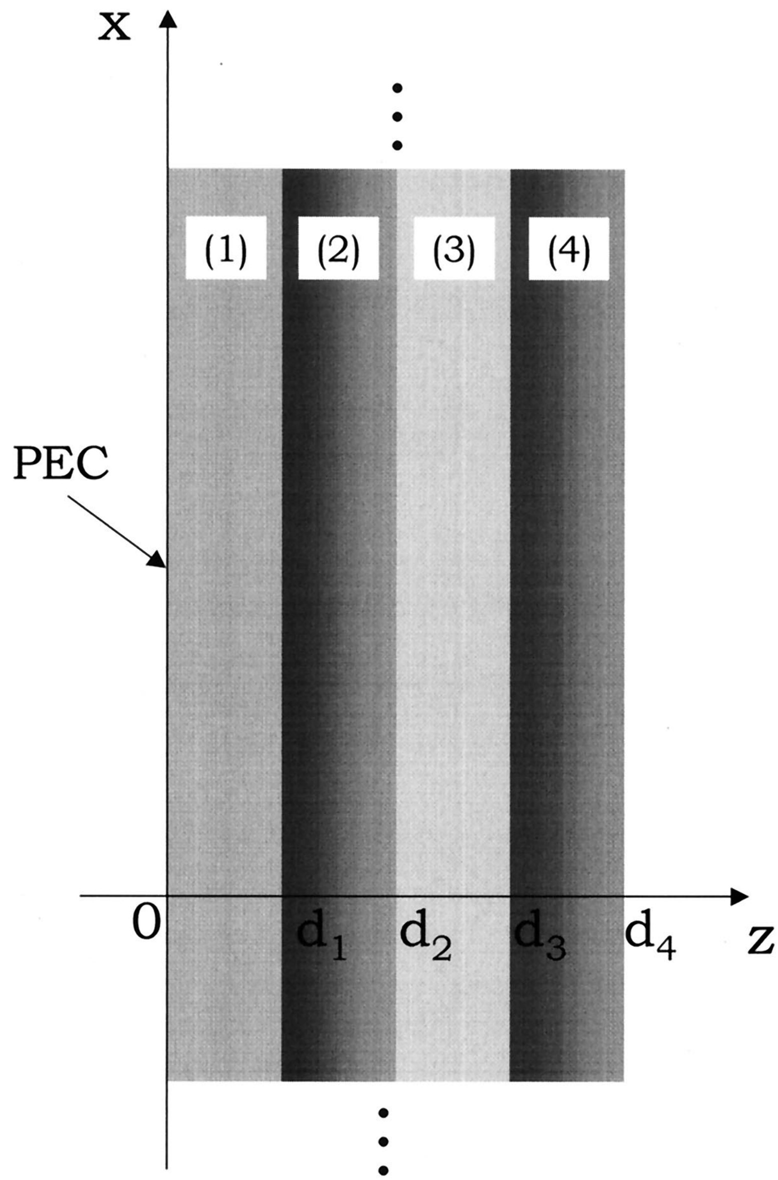


Figure 5. Four-layer coating. For layer 1 (gyrotropic), $\varepsilon_r = 10$, $\mu_{ra} = 5 - i0.3$, $\mu_{rb} = 2 - i0.1$, and $l_1 = d_1 = 0.025\lambda_0$. For layer 2 (parabolic-profile dielectric), $\varepsilon_r^{(d_1)} = 10$, $\varepsilon_r^{(d_2)} = 5$, $\mu_r = 1$, and $l_2 = d_2 - d_1 = 0.025\lambda_0$. For layer 3 (chiral), $\varepsilon_r = 5 - i0.2$, $\mu_r = 2$, $\gamma_c = 0.005\ \Omega^{-1}$, and $l_3 = d_3 - d_2 = 0.025\lambda_0$. For layer 4 (linear-profile dielectric), $\varepsilon_r^{(d_3)} = 5$, $\varepsilon_r^{(d_4)} = 2$, $\mu_r = 1$, and $l_4 = d_4 - d_3 = 0.025\lambda_0$.

where

$$[M_E(k_x)] = \begin{bmatrix} 0 & k_{z0}/k_0 \\ 1 & 0 \end{bmatrix},$$
$$[M_H(k_x)] = \frac{1}{\eta_0}\begin{bmatrix} k_{z0}/k_0 & 0 \\ 0 & -1 \end{bmatrix}, \qquad k_{z0} = \sqrt{k_0^2 - k_x^2}. \tag{45}$$

Figure 4 displays the magnitude and phase of the reflection coefficients as functions of the incidence angle ($\theta_i = \sin^{-1}(k_x/k_0)$), computed using exact IBCs, second-order IBCs, and the TIBC (normal incidence approximation, i.e., $k_x = 0$). As one can see, second-order IBCs give very accurate results. Conversely, TIBC does not predict any cross-polar-

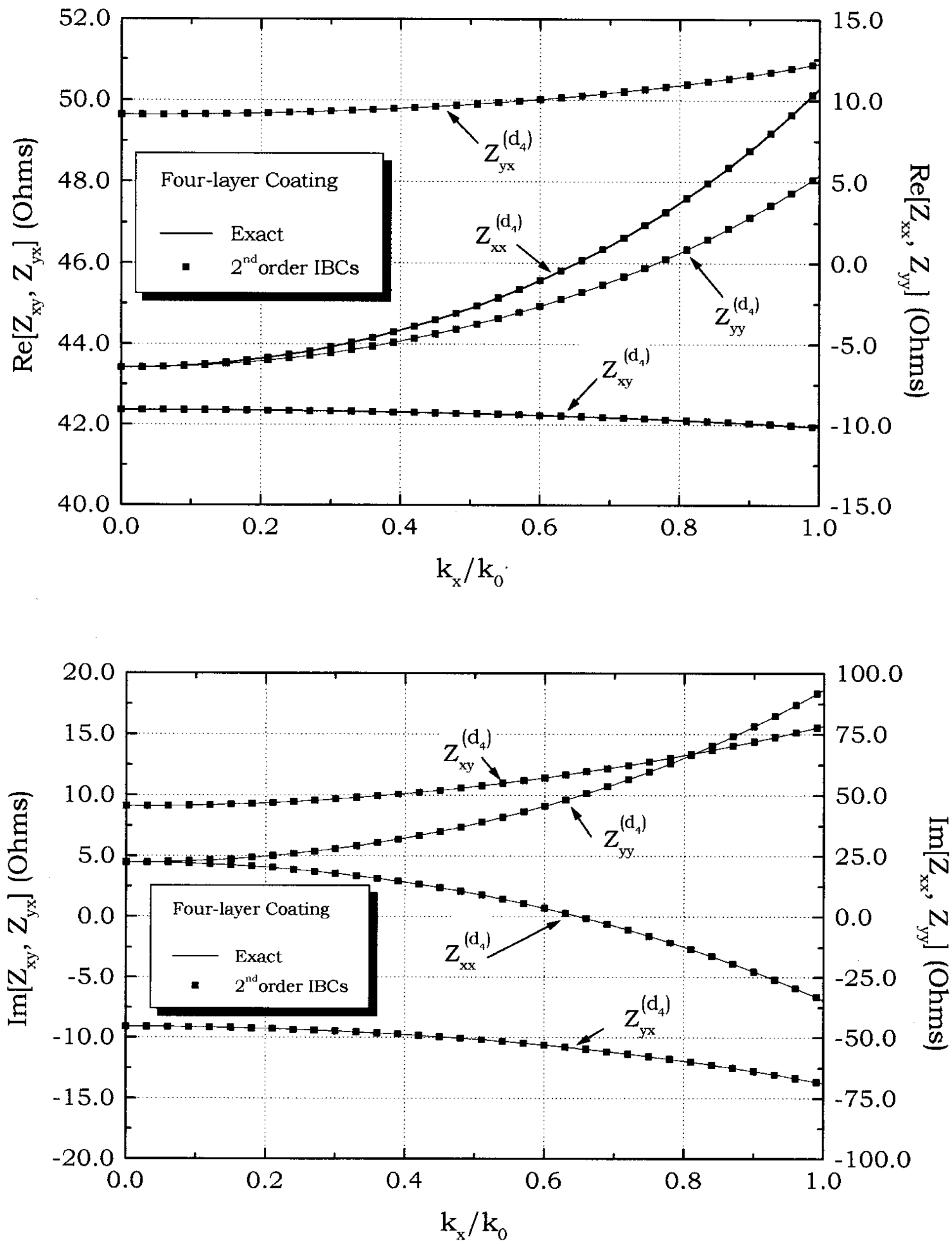


Figure 6. IBCs for the four-layer coating of Figure 5.

ized reflection ($R_{\mathrm{TETM}} = R_{\mathrm{TMTE}} = 0$, $|R_{\mathrm{TETE}}| = |R_{\mathrm{TMTM}}| = 1$), since chirality does not affect normal incidence. TIBC also fails in predicting the phase behavior of the copolarized reflection components.

As a further example, we consider a nonreciprocal, lossy, four-layer coating on a perfectly conducting plane, as depicted in Figure 5. It consists of (1) a lossy, homogeneous gyrotropic layer, (2) a lossless, parabolic-profile dielectric layer, (3) a lossy, homogeneous chiral layer, and (4) a lossless, linear-profile dielectric layer. Similar complex coatings are likely found in modern broadband absorbers.

The gyrotropic material is described by the following constitutive relations [*Kong*, 1975]:

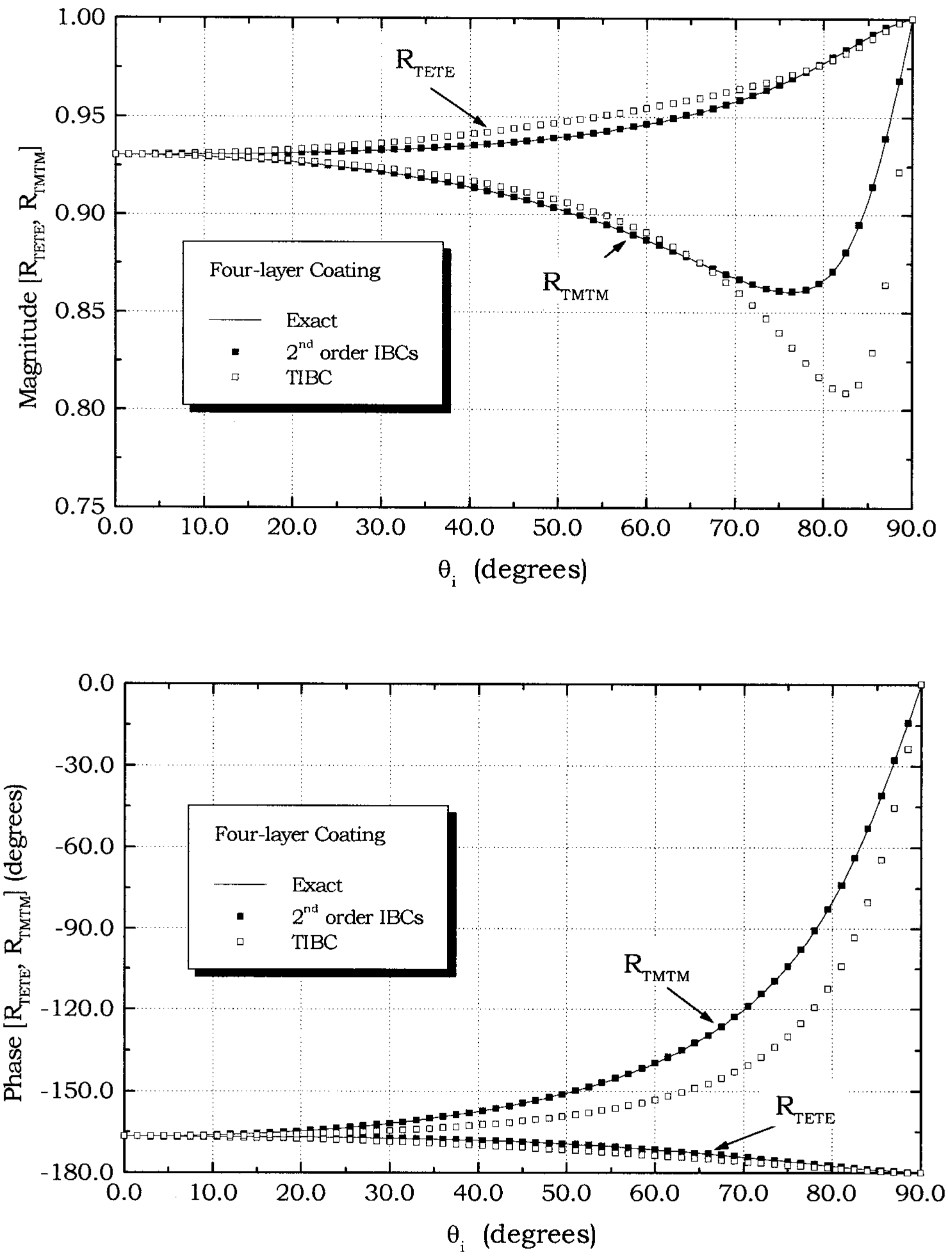


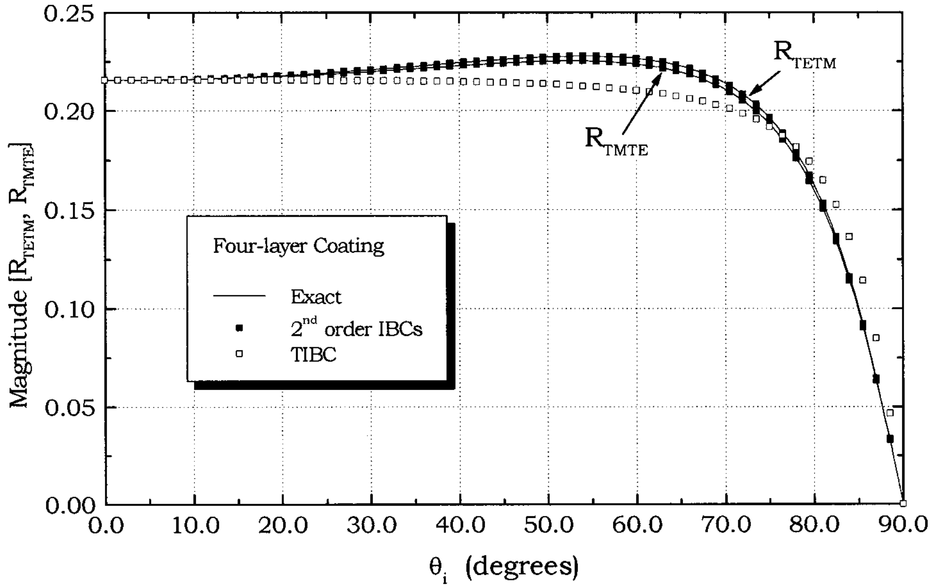
Figure 7. Copolar plane wave reflection components for the four-layer coating of Figure 5.

$$\begin{aligned}\mathbf{D} &= \varepsilon_0 \varepsilon_r \mathbf{E},\\ \mathbf{B} &= \mu_0 \bar{\mu}_{rg} \mathbf{H},\end{aligned} \qquad \bar{\mu}_{rg} = \begin{bmatrix} \mu_{ra} & i\mu_{rb} & 0 \\ -i\mu_{rb} & \mu_{ra} & 0 \\ 0 & 0 & \mu_{ra} \end{bmatrix}, \tag{46}$$

and the corresponding (transverse) characteristic waves and propagation constants are reported in Appendix D. The parabolic-profile dielectric layer is described by

$$\varepsilon_r(z) = \varepsilon_r^{(d_1)} \left[1 - \frac{(z - d_1)^2 (\varepsilon_r^{(d_1)} - \varepsilon_r^{(d_2)})}{\varepsilon_r^{(d_1)} (d_2 - d_1)^2} \right], \qquad d_1 < z < d_2 \; . \tag{47}$$

HOIBCs for this multilayer coating may be derived by simply iterating the procedure described in section 2. The results are shown in Figure 6 (exact and



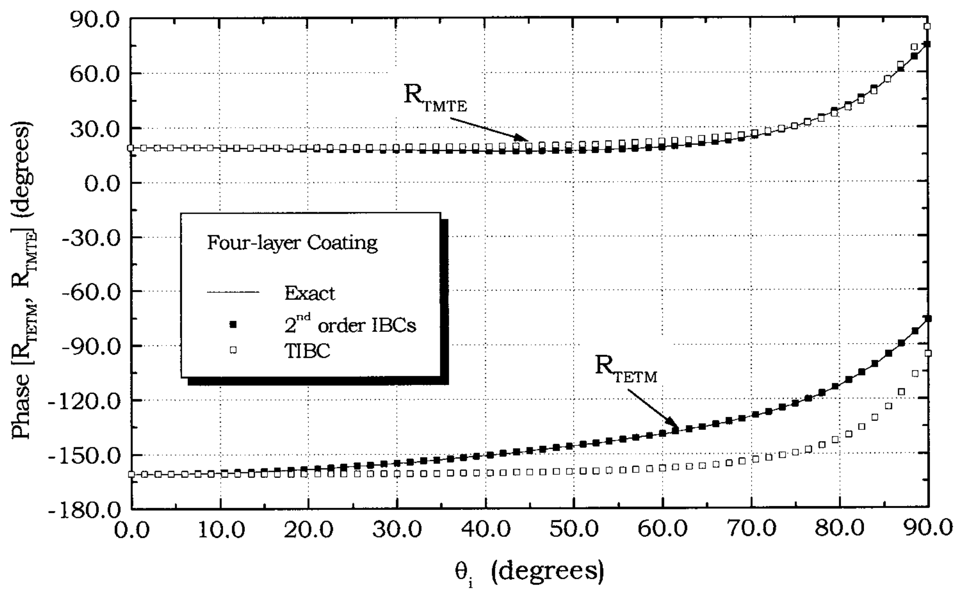


Figure 8. Cross-polar plane wave reflection components for the four-layer coating of Figure 5.

approximate impedance terms) and Figures 7 and 8 (reflection coefficients). Again, it can be observed that second-order IBCs do a fine job, whereas TIBC gives considerably worse results.

Obviously, the accuracy provided by the proposed HOIBCs depends on the coating parameters. This issue has been thoroughly investigated in the case of dielectric multilayers by *Hoppe and Rahmat-Samii* [1995] and *Galdi and Pinto* [1999a]. In particular, it has been pointed out that the proposed framework is actually capable of modeling even the presence of surface waves in the coating. (*Hoppe and Rahmat-Samii* [1995] also addressed the problem of solving the transverse-resonance equation by using rational-approximated spectral impedances.) It has been observed that as the (average) electrical length of the coating approaches a quarter of a wavelength, the first pole in k_x space may appear. Although their rational structure potentially enables the proposed second-order IBCs to model the effects of pole singularities, in fact the accuracy provided gets usually poorer, and hence IBCs of higher order are required. Generalization of the above to the case of bianisotropic layers, though possible in principle, is less handy owing to the presence of more constitutive parameters.

We point out that the problem of deriving this kind of IBC is conceptually amenable to the so-called model-based parameter estimation, namely, given a known function (spectral impedances, in our case) and a fitting model (rational functions, in our case), finding out the unknown model parameters (order and coefficients of the rational approximants, in our case) to fulfill some optimality criteria. In this connection, a discussion of the main issues concerning accuracy assessment is given by *Miller and Burke* [1989] and *Miller* [1996].

4. Conclusions and Recommendations

A systematic spectral domain approach for deriving HOIBCs describing stratified planar coatings consisting of an arbitrary number of homogeneous bianisotropic and inhomogeneous dielectric layers on a general (polarization-rotating) impedance plane has been presented. The proposed framework extends the approach formulated by *Hoppe and Rahmat-Samii* [1995] for homogeneous coatings. Computational examples indeed show that fairly complex coatings can be easily handled with accurate results by using second-order IBCs.

The proposed HOIBCs turn out to be profitably implemented in computer-aided design tools for design of reduced-reflection coatings, printed antennas, shields, radomes, etc. In particular, they allow a rather easy modeling of two-dimensional (multilayer) coated bodies and, possibly, bodies of revolution [*Hoppe and Rahmat-Samii*, 1995], with (local) curvature radii comparable to (or larger than) a wavelength and slowly varying geometrical/constitutive properties. It has been pointed out that handling geometrical/constitutive discontinuities (e.g., grooves, wedges) is possible at the expense of introducing additional constraints.

HOIBCs relying on locally cylindrical approximation, which allows an accurate modeling of curvature effects, have been recently worked out and are the subject of a forthcoming paper [*Galdi and Pinto*, 1999b]. Worthwhile extensions, presently under investigation, include inhomogeneous bianisotropic layers and two-dimensional curvature effects.

Appendix A: IBCs for Bianisotropic Layers

By separating into pairs the four unknown coefficients, the tangential fields can be written in matrix form as [*Hoppe and Rahmat-Samii*, 1995]

$$\begin{bmatrix} E_x(k_x, z) \\ E_y(k_x, z) \end{bmatrix} = [E_{12}(k_x, z)] \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + [E_{34}(k_x, z)] \cdot \begin{bmatrix} c_3 \\ c_4 \end{bmatrix}, \tag{A1}$$

$$\begin{bmatrix} H_x(k_x, z) \\ H_y(k_x, z) \end{bmatrix} = [H_{12}(k_x, z)] \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + [H_{34}(k_x, z)] \cdot \begin{bmatrix} c_3 \\ c_4 \end{bmatrix}, \tag{A2}$$

where $[E_{pq}]$, $[H_{pq}]$ are defined by (10) and (11). By enforcing the boundary conditions (7) at the ground plane ($z = 0$), the coefficients c_3, c_4 can be determined in terms of c_1, c_2, namely,

$$\begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = [M^{(0)}(k_x)] \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}, \tag{A3}$$

where $[M^{(0)}]$ is defined by (9). The IBCs (8) are finally obtained by eliminating the coefficients c_1, c_2 in (A1) and (A2).

Appendix B: Coefficient Determination in Equations (35)–(37)

By enforcing (32) at $k_x = 0$, $k_x^{(1)}$, $k_x^{(2)}$, one gets [*Hoppe and Rahmat-Samii*, 1995]

$$\begin{aligned} c_0^{(5)} &= Z_{xx}^{(d)}(0) = Z_{yy}^{(d)}(0), \\ c_0^{(6)} &= Z_{xy}^{(d)}(0) = -Z_{xy}^{(d)}(0), \end{aligned} \tag{B1}$$

$$\begin{bmatrix} c_2^{(1)} \\ c_2^{(2)} \\ c_2^{(5)} \\ c_2^{(6)} \end{bmatrix} = [M]^{-1} \cdot \begin{bmatrix} Z_{xx}^{(d)}(0) - Z_{xx}^{(d)}(k_x^{(1)}) \\ Z_{xy}^{(d)}(0) - Z_{xy}^{(d)}(k_x^{(1)}) \\ Z_{xx}^{(d)}(0) - Z_{xx}^{(d)}(k_x^{(2)}) \\ Z_{xy}^{(d)}(0) - Z_{xy}^{(d)}(k_x^{(2)}) \end{bmatrix}, \qquad \text{(B2)}$$

$$\begin{bmatrix} c_2^{(3)} \\ c_2^{(4)} \\ c_2^{(7)} \\ c_2^{(8)} \end{bmatrix} = [M]^{-1} \cdot \begin{bmatrix} Z_{yx}^{(d)}(0) - Z_{yx}^{(d)}(k_x^{(1)}) \\ Z_{yy}^{(d)}(0) - Z_{yy}^{(d)}(k_x^{(1)}) \\ Z_{yx}^{(d)}(0) - Z_{yx}^{(d)}(k_x^{(2)}) \\ Z_{yy}^{(d)}(0) - Z_{yy}^{(d)}(k_x^{(2)}) \end{bmatrix}, \qquad \text{(B3)}$$

where

$$[M] = \begin{bmatrix} k_x^{(1)^2} Z_{xx}^{(d)}(k_x^{(1)}) & k_x^{(1)^2} Z_{yx}^{(d)}(k_x^{(1)}) & -k_x^{(1)^2} & 0 \\ k_x^{(1)^2} Z_{xy}^{(d)}(k_x^{(1)}) & k_x^{(1)^2} Z_{yy}^{(d)}(k_x^{(1)}) & 0 & -k_x^{(1)^2} \\ k_x^{(2)^2} Z_{xx}^{(d)}(k_x^{(2)}) & k_x^{(2)^2} Z_{yx}^{(d)}(k_x^{(2)}) & -k_x^{(2)^2} & 0 \\ k_x^{(2)^2} Z_{xy}^{(d)}(k_x^{(2)}) & k_x^{(2)^2} Z_{yy}^{(d)}(k_x^{(2)}) & 0 & -k_x^{(2)^2} \end{bmatrix}. \qquad \text{(B4)}$$

Appendix C: Transverse Characteristic Waves for a Chiral Medium

It is readily found from (3), (4), and (6) that

$$\mathbf{e}_t^{(1)}(k_x) = i\,\frac{k_z^{(1)}}{k_r}\,\hat{i}_x + \hat{i}_y, \quad \mathbf{h}_t^{(1)}(k_x) = \frac{i}{\eta_c}\,\mathbf{e}_t^{(1)}(k_x), \qquad \text{(C1)}$$

$$k_z^{(1)} = \sqrt{k_r^2 - k_x^2},$$

$$\mathbf{e}_t^{(2)}(k_x) = i\,\frac{k_z^{(2)}}{k_r}\,\hat{i}_x + \hat{i}_y, \quad \mathbf{h}_t^{(2)}(k_x) = \frac{i}{\eta_c}\,\mathbf{e}_t^{(2)}(k_x), \qquad \text{(C2)}$$

$$k_z^{(2)} = -\sqrt{k_r^2 - k_x^2},$$

$$\mathbf{e}_t^{(3)}(k_x) = -i\,\frac{k_z^{(3)}}{k_l}\,\hat{i}_x + \hat{i}_y, \quad \mathbf{h}_t^{(3)}(k_x) = -\frac{i}{\eta_c}\,\mathbf{e}_t^{(3)}(k_x), \qquad \text{(C3)}$$

$$k_z^{(3)} = \sqrt{k_l^2 - k_x^2},$$

$$\mathbf{e}_t^{(4)}(k_x) = -i\,\frac{k_z^{(4)}}{k_l}\,\hat{i}_x + \hat{i}_y, \quad \mathbf{h}_t^{(4)}(k_x) = -\frac{i}{\eta_c}\,\mathbf{e}_t^{(4)}(k_x), \qquad \text{(C4)}$$

$$k_z^{(4)} = -\sqrt{k_l^2 - k_x^2},$$

$$\left.\begin{matrix} k_r \\ k_l \end{matrix}\right\} = k_0\sqrt{\mu_r(\varepsilon_r + \mu_r\eta_0^2\gamma_c^2)} \pm k_0\eta_0\gamma_c,$$

$$\eta_c = \eta_0\sqrt{\frac{\mu_r}{\varepsilon_r + \mu_r\eta_0^2\gamma_c^2}}. \qquad \text{(C5)}$$

(Note that similar equations of *Hoppe and Rahmat-Samii* [1995] are affected by typographical errors.)

Appendix D: Transverse Characteristic Waves for a Gyrotropic Medium

It is readily found from (3), (4), and (6) that

$$\mathbf{e}_t^{(1)}(k_x) = -i\,\frac{k_{za}}{k_0\sqrt{\varepsilon_r\mu_{ra}}}\,\hat{i}_x + \hat{i}_y,$$

$$\mathbf{h}_t^{(1)}(k_x) = -\frac{k_{za}^2}{k_z^{(1)}k_0\eta_0\mu_{ra}}\,\hat{i}_x - i\,\frac{\sqrt{\varepsilon_r}\,k_{za}}{k_z^{(1)}\eta_0\sqrt{\mu_{ra}}}\,\hat{i}_y, \qquad \text{(D1)}$$

$$\mathbf{e}_t^{(2)}(k_x) = i\,\frac{k_{za}}{k_0\sqrt{\varepsilon_r\mu_{ra}}}\,\hat{i}_x + \hat{i}_y,$$

$$\mathbf{h}_t^{(2)}(k_x) = -\frac{k_{za}^2}{k_z^{(2)}k_0\eta_0\mu_{ra}}\,\hat{i}_x + i\,\frac{\sqrt{\varepsilon_r}\,k_{za}}{k_z^{(2)}\eta_0\sqrt{\mu_{ra}}}\,\hat{i}_y, \qquad \text{(D2)}$$

$$\mathbf{e}_t^{(3)}(k_x) = -i\,\frac{k_{za}}{k_0\sqrt{\varepsilon_r\mu_{ra}}}\,\hat{i}_x + \hat{i}_y,$$

$$\mathbf{h}_t^{(3)}(k_x) = -\frac{k_{za}^2}{k_z^{(3)}k_0\eta_0\mu_{ra}}\,\hat{i}_x - i\,\frac{\sqrt{\varepsilon_r}\,k_{za}}{k_z^{(3)}\eta_0\sqrt{\mu_{ra}}}\,\hat{i}_y, \qquad \text{(D3)}$$

$$\mathbf{e}_t^{(4)}(k_x) = i\,\frac{k_{za}}{k_0\sqrt{\varepsilon_r\mu_{ra}}}\,\hat{i}_x + \hat{i}_y,$$

$$\mathbf{h}_t^{(4)}(k_x) = -\frac{k_{za}^2}{k_z^{(4)}k_0\eta_0\mu_{ra}}\,\hat{i}_x + i\,\frac{\sqrt{\varepsilon_r}\,k_{za}}{k_z^{(4)}\eta_0\sqrt{\mu_{ra}}}\,\hat{i}_y, \qquad \text{(D4)}$$

$$k_z^{(1)} = \sqrt{k_{za}^2 + \frac{k_0k_{za}\sqrt{\varepsilon_r}\,\mu_{rb}}{\sqrt{\mu_{ra}}}},$$

$$k_z^{(2)} = \sqrt{k_{za}^2 - \frac{k_0k_{za}\sqrt{\varepsilon_r}\,\mu_{rb}}{\sqrt{\mu_{ra}}}}, \qquad \text{(D5)}$$

$$k_z^{(3)} = -\sqrt{k_{za}^2 + \frac{k_0k_{za}\sqrt{\varepsilon_r}\,\mu_{rb}}{\sqrt{\mu_{ra}}}},$$

$$k_z^{(4)} = -\sqrt{k_{za}^2 - \frac{k_0k_{za}\sqrt{\varepsilon_r}\,\mu_{rb}}{\sqrt{\mu_{ra}}}}, \qquad \text{(D6)}$$

$$k_{za} = \sqrt{k_0^2 \varepsilon_r \mu_{ra} - k_x^2}\,. \qquad \text{(D7)}$$

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