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# Electromagnetic funneling through a single-negative slab paired with a double-positive transformation slab

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## Abstract

**Purpose** – The work is aimed at studying the electromagnetic interaction between a homogeneous, isotropic single-negative (SNG) slab and an inhomogeneous, anisotropic double-positive (DPS) slab.

**Design/methodology/approach** – The approach is based on the transformation optics framework, which allows systematic design and modelling of anisotropic, inhomogeneous metamaterials with inherent field-manipulation capabilities.

**Findings** – The paper finds that a transformation-optics-based DPS slab can compensate the inherent opaqueness to the electromagnetic radiation of a SNG slab. Here, "compensation" means that the resulting bi-layer may give rise to zero-reflection for a normally-incident plane wave at a given frequency. Such phenomenon is inherently accompanied by (de)funneling effects for collimated-beam illumination, and it turns out to be quite robust to material losses as well as geometrical and constitutive-parameter truncation.

**Originality/value** – The results provide further evidence and insight in how SNG-like responses may be emulated (within narrow parametric ranges) by suitably-engineered spatial inhomogeneity and anisotropy in DPS media. Moreover, they also show that resonant transmission phenomena through SNG materials may be engineered via the powerful framework of transformation optics.

**Keywords** Transformation optics, Metamaterials, Funneling, Optics, Electromagnetism, Materials

**Paper type** Research paper

## 1. Introduction

In single-negative (SNG) media, like epsilon-negative (ENG) and mu-negative (MNG) materials, the propagation constant exhibits a dominant imaginary character, thereby resulting in an evanescent decay of electromagnetic (EM) waves. In spite of this inherent opaqueness, very interesting, and sometime counterintuitive, effects may be obtained via suitable arrangements and combinations of such materials.

For instance, it was shown (Alù and Engheta, 2003) that pairing, under suitable matching conditions, two planar slabs made of ENG and MNG materials, respectively, perfect electromagnetic tunneling (with zero phase delay) may be achieved, mediated by

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the excitation of a surface plasmon polariton at the interface. This intriguing and counterintuitive (in view of the opaqueness of the single constituents taken alone) result motivated a growing interest in the transmission properties of heterostructures containing SNG materials, possibly combined with other type of materials, with a wide variety of extensions and generalizations proposed in the recent literature (Zhou *et al.*, 2005; Hou *et al.*, 2005; Guan *et al.*, 2006; Fang and He, 2008; Kim and Lee, 2008; Feng *et al.*, 2009; Oraizi and Abdolali, 2009; Castaldi *et al.*, 2010, 2011a, b; Ding *et al.*, 2010; Butler *et al.*, 2011; Cojocaru, 2011).

Particularly worth of mention are the configurations in Zhou *et al.* (2005), Hou *et al.* (2005), Oraizi and Abdolali (2009), Castaldi *et al.* (2010, 2011a), Butler *et al.* (2011) and Cojocaru (2011) featuring SNG materials combined with double-positive (DPS) materials. More specifically, in Zhou *et al.* (2005), Hou *et al.* (2005) and Butler *et al.* (2011), it was shown numerically and experimentally (at microwave frequencies) that, for an ENG slab sandwiched symmetrically between two identical DPS layers, total transmission (with phase delay) may occur under normal-incidence illumination.

In a series of recent investigations (Castaldi *et al.*, 2010, 2011a), we have considered similar configurations, but with asymmetrical geometries, i.e. with the DPS layer(s) at one side only. In particular, we considered an ENG slab paired with a bi-layer of homogeneous DPS materials (Castaldi *et al.*, 2010) under normal-incidence illumination, or with a homogeneous, uniaxial DPS layer under oblique-incidence illumination (Castaldi *et al.*, 2011a). For given (frequency, direction) illumination conditions, we were able to derive analytically the design rules for achieving total transmission, and showed that the DPS constituent(s) may effectively act as an equivalent "matched" MNG slab over relatively narrow frequency and angular ranges.

Expanding upon the above studies, we explore here another anomalous transmission mechanism which entails pairing a homogeneous, isotropic SNG slab with an inhomogeneous, anisotropic DPS slab based on the transformation-optics paradigm (Leonhardt, 2006; Pendry *et al.*, 2006), which has recently emerged as a promising approach for the systematic design of metamaterials with inherent field-manipulation capabilities (Chen *et al.*, 2010). Accordingly, the rest of the paper is laid out as follows. In Section 2, we outline the problem geometry and formulation. In Section 3, we present our analytical derivations and physical interpretation of the underlying phenomena. In Section 4, we discuss some representative numerical results and, in connection with a collimated-beam excitation, we highlight the arising (de)funneling effects. Finally, in Section 5, we provide some conclusions and hints for future research.

## 2. Problem geometry and formulation

The geometry of interest is shown in Figure 1. Without loss of generality, in a vacuum background (i.e. relative permittivity and permeability $\varepsilon = \mu = 1$), we consider a homogeneous, isotropic, nonmagnetic ($\mu_1 = 1$) ENG slab of thickness $d_1$ and relative permittivity $\varepsilon_1$ (with $\mathrm{Re}(\varepsilon_1) < 0$) paired with a slab of thickness $d_2$ made of an inhomogeneous, anisotropic "transformation medium" described by the relative permittivity and permeability tensors:

$$\underline{\underline{\varepsilon_2}}(x,y) = \underline{\underline{\mu_2}}(x,y) = \det\left[\underline{\underline{J}}(x,y)\right]\underline{\underline{J}}^{-1}(x,y)\cdot\underline{\underline{J}}^{-T}(x,y), \tag{1}$$

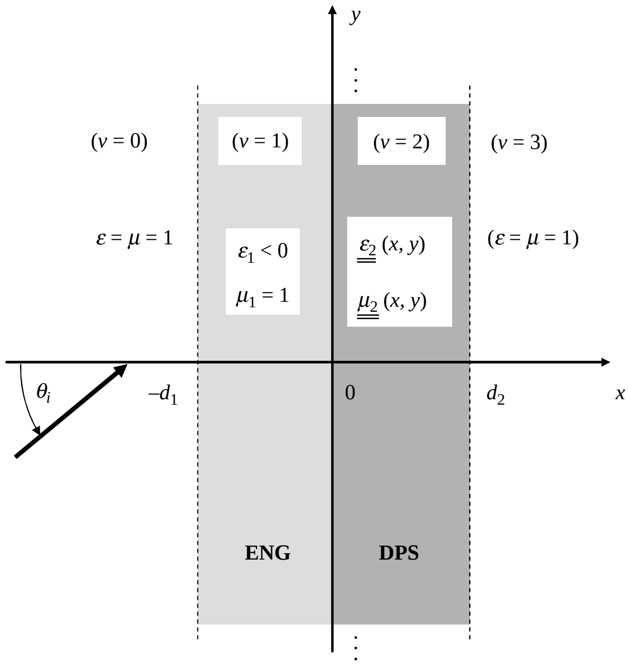



**Figure 1.**
Problem geometry (details in the text)

under time-harmonic ($\exp(-i\omega t)$), obliquely-incident, transverse-magnetic (TM) plane-wave illumination, with unit-amplitude, $z$-directed magnetic field. Within the framework of transformation optics (Pendry *et al.*, 2006), the medium in equation (1) embeds, via the Jacobian matrix $\underline{\underline{J}} \equiv \partial(x', y', z')/\partial(x, y, z)$ (with $-T$ denoting the inverse transpose), the effects of a two-dimensional (2D) coordinate transformation between a fictitious (vacuum) space ($x'$, $y'$, $z'$) and the slab region $0 < x < d_2$ in the actual physical space ($x$, $y$, $z$). In particular, we focus here on a class of transformation media arising from the following coordinate transformations (Gallina *et al.*, 2010):

$$\begin{cases} x' = u(x), \\ y' = \dfrac{y}{\dot{u}(x)}, \\ z' = z, \end{cases} \tag{2}$$

where $u(x)$ is real, differentiable function, the overdot indicates differentiation with respect to the argument, and the assumption $\dot{u}(x) > 0$ is made in order to avoid singularities. Following (Gallina *et al.*, 2010), it can be shown that the constitutive tensors in equation (1) are always positive-defined, i.e. representative of a DPS medium which, for the assumed TM polarization, is effectively nonmagnetic (i.e. $\mu_{2z} = 1$).

In what follows, we address the analytical solution of the above problem and, for given illumination conditions, we derive the conditions (on the mapping function $u$ in equation (2)) under which the ENG-DPS bi-layer exhibits zero-reflection.

## 3. Analytical results

### 3.1 General solution

A general analytical solution for the problem of interest can be systematically derived by following a procedure similar to that in our earlier work (Gallina *et al.*, 2010). We start considering the solution in an auxiliary space $(x', y', z')$ containing the ENG slab only (embedded in vacuum), under the assumed TM-polarized, obliquely-incident, plane-wave illumination:

$$H_z^i(x', y') = \exp[i(k_{x0}x' + k_{y0}y')], \tag{3}$$

which can easily be derived analytically. In equation (3), assuming a propagating wave (Figure 1):

$$k_{x0} = k_0 \cos\theta_i, \quad k_{y0} = k_0 \sin\theta_i, \tag{4}$$

with $k_0 = \omega/c_0 = 2\pi/\lambda_0$ denoting the vacuum wavenumber (and $c_0$, $\lambda_0$ the corresponding speed of light and wavelength, respectively). Along the lines of (Gallina *et al.*, 2010), the solution in the actual physical space $(x, y, z)$ can be compactly written as:

$$\begin{aligned} H_z^{(\nu)}(x,y) = {} & A_\nu^+ \exp[ik_{x\nu}x'(x,y) + ik_{y\nu}y'(x,y)] \\ & + A_\nu^- \exp[-ik_{x\nu}x'(x,y) + ik_{y\nu}y'(x,y)], \quad \nu = 0,1,2,3, \end{aligned} \tag{5}$$

with reference to the four $\nu$-indexed regions identified in Figure 1. In equation (5), the coordinate mapping is assumed to be the identity ($x' = x, y' = y$) everywhere except for the region $0 < x < d_2$ ($\nu = 2$) occupied by the transformation slab, for which it is given by equation (2). Enforcing the tangential-field continuity and boundary conditions at the interfaces $x = -d_1$, $x = 0$, and $x = d_2$ yields the unknown amplitude coefficients $A_\nu^\pm$ as well as the wavenumbers $k_{x\nu}$ and $k_{y\nu}$. In particular, the assumed unit-amplitude excitation and the radiation condition readily yield $A_0^+ = 1$ and $A_3^- = 0$, respectively, whereas enforcement of the phase-matching conditions at the interfaces $x = -d_1, x = 0$, and $x = d_2$ allows to relate the wavenumbers in regions $\nu = 1,2,3$ to those pertaining to the incident plane wave in equation (4), namely:

$$k_{y1} = k_{y0}, \quad k_{y2} = k_{y0}\dot{u}(0), \quad k_{y3} = k_{y0}\frac{\dot{u}(0)}{\dot{u}(d_2)}, \tag{6}$$

$$k_{x1} = i\sqrt{k_{y0}^2 - k_0^2\varepsilon_1} = i\alpha_x, \quad \mathrm{Im}(k_{x1}) \geq 0, \tag{7}$$

$$k_{x\nu} = \sqrt{k_0^2 - k_{y\nu}^2}, \quad \mathrm{Im}(k_{x\nu}) \geq 0, \quad \nu = 2,3. \tag{8}$$

Finally, the remaining six amplitude coefficients $A_0^-$, $A_1^+$, $A_1^-$, $A_2^+$, $A_2^-$, and $A_3^+$ are computed by enforcing the continuity of the tangential fields (with the electric field obtained from equation (5) via the relevant Maxwell's curl equation). In particular, we focus here on the expression of the reflected-field amplitude:

$$A_0^- = \left[\frac{i\chi_0 + \chi_1 \tanh(\alpha_x d_1) + \chi_2 \tan\kappa_{20} + i\chi_{12}\tanh(\alpha_x d_1)\tan\kappa_{20}}{i\xi_0 + \xi_1 \tanh(\alpha_x d_1) + \xi_2 \tan\kappa_{20} + i\xi_{12}\tanh(\alpha_x d_1)\tan\kappa_{20}}\right] \exp\,(-ik_{x0}d_1), \tag{9}$$

where:

$$\kappa_2 = k_{x2}[u(d_2) - u(0)], \tag{10}$$

$$\chi_0 = \alpha_x k_{x2} \varepsilon_1 [k_{x3}\dot{u}(d_2) - k_{x0}\dot{u}(0)], \quad \chi_1 = -k_{x2}\left[\varepsilon_1^2 k_{x0} k_{x3} \dot{u}(d_2) + \alpha_x^2 \dot{u}(0)\right],$$
$$\chi_2 = \alpha_x \varepsilon_1 \left[k_{x2}^2 - k_{x0} k_{x3} \dot{u}(0)\dot{u}(d_2)\right], \quad \chi_{12} = \varepsilon_1^2 k_{x0} k_{x2}^2 + \alpha_x^2 k_{x3} \dot{u}(0)\dot{u}(d_2), \tag{11}$$

$$\xi_0 = -\alpha_x k_{x2} \varepsilon_1 [k_{x3}\dot{u}(d_2) + k_{x0}\dot{u}(0)], \quad \xi_1 = k_{x2}\left[\alpha_x^2 \dot{u}(0) - \varepsilon_1^2 k_{x0} k_{x3} \dot{u}(d_2)\right],$$
$$\xi_2 = -\varepsilon_1 \alpha_x \left[k_{x2}^2 + k_{x0} k_{x3} \dot{u}(0)\dot{u}(d_2)\right], \quad \xi_{12} = \varepsilon_1^2 k_{x0} k_{x2}^2 - \alpha_x^2 k_{x3} \dot{u}(0)\dot{u}(d_2). \tag{12}$$

*3.2 Zero-reflection conditions*

Assuming (subject to an *a posteriori* verification) that the denominator of equation (9) is nonzero, the conditions for zero-reflection of the ENG-DPS bi-layer can be determined by zeroing the numerator, namely:

$$i\chi_0 + \chi_1 \tanh(\alpha_x d_1) + \chi_2 \tan \kappa_2 + i\chi_{12} \tan \kappa_2 \tanh(\alpha_x d_1) = 0, \tag{13}$$

which depends on the ENG slab parameters, as well as on the frequency, incidence direction, and boundary values of the mapping function and its derivative. Assuming normal incidence ($\theta_i = 0$) and ideal lossless conditions, the wavenumbers in equations (6)-(8) simplify as:

$$k_{y1} = k_{y2} = k_{y3} = k_{y0} = 0, \quad k_{x2} = k_{x3} = k_{x0} = k_0, \quad \alpha_x = \sqrt{-\varepsilon_1} k_0 > 0. \tag{14}$$

Under these conditions, zeroing the real part of equation (13), and recalling equation (11), we obtain:

$$\kappa_2 = \arctan\left[\frac{[\varepsilon_1 \dot{u}(d_2) - \dot{u}(0)] \tanh(\alpha_x d_1)}{\sqrt{-\varepsilon_1}[1 - \dot{u}(0)\dot{u}(d_2)]}\right] + m\pi > 0. \tag{15}$$

From equation (10), in view of the upfront assumption $\dot{u}(x) > 0$, it follows that $\kappa_2 > 0$, and thus the (integer) value of $m$ in equation (15) is chosen so as to guarantee the smallest positive result. Substituting equation (15) in equation (13) and zeroing the remaining imaginary part yields:

$$(\varepsilon_1 w - 1)[\varepsilon_1 - w\dot{u}^2(0)] \tanh^2(\alpha_x d_1) = \varepsilon_1 (w - 1)[1 - w\dot{u}^2(0)], \tag{16}$$

which, having defined the auxiliary parameter:

$$w = \frac{\dot{u}(d_2)}{\dot{u}(0)}, \tag{17}$$

is a simple quadratic equation:

$$\dot{u}^2(0) = \frac{\varepsilon_1 \left[w - 1 + (1 - \varepsilon_1 w) \tanh^2(\alpha_x d_1)\right]}{w\left[\varepsilon_1 (w - 1) + (1 - \varepsilon_1 w) \tanh^2(\alpha_x d_1)\right]}, \tag{18}$$

that admits real solutions in $\dot{u}(0)$ if:

$$w < \frac{1 - \tanh^2(\alpha_x d_1)}{1 - \varepsilon_1 \tanh^2(\alpha_x d_1)} < 1, \tag{19}$$

or:

$$w > \frac{\tanh^2(\alpha_x d_1) - \varepsilon_1}{-\varepsilon_1 \left[1 - \tanh^2(\alpha_x d_1)\right]} > 1. \tag{20}$$

From the above conditions, it clearly emerges that the coordinate transformation in equation (2) cannot be the identity (i.e. $w = 1$), and thus, as also elucidated in Zhou *et al.* (2005) and Kim and Lee (2008), it is impossible to attain total zero-reflection by pairing homogeneous, isotropic ENG and DPS slabs in vacuum. On the other hand, for given parameters of the ENG slab ($\varepsilon_1$, $d_1$), it is always possible to design a DPS transformation slab (i.e. mapping function $u$ and thickness $d_2$) that, paired with the ENG slab, yields zero-reflection at a prescribed frequency. In what follows, for a given choice of the parameter $w \neq 1$, we focus on the possibly simplest conceivable (quadratic) mapping:

$$u(x) = U_0 \left[ \left( \frac{w-1}{2d_2} \right) x^2 + x \right], \tag{21}$$

with $U_0 = \dot{u}(0)$ given by equation (18). This completely defines (together with a proper choice of the integer $m$) the right-hand-side in equation (15), which, recalling equation (10), yields a linear equation:

$$\kappa_2 = \frac{k_0(w+1)d_2}{2}, \tag{22}$$

from which we can readily compute the thickness $d_2$. It is worth emphasizing that the constraints in equations (19) and (20) imply that the larger the permittivity (absolute value) and thickness of the ENG slab, the more pronounced (up to extreme values) the anisotropy of the arising transformation medium.

*3.3 Transmitted field*

Under the above derived zero-reflection conditions, the excitation coefficient of the field transmitted through the ENG-DPS bi-layer is given by:

$$A_3^+ = \frac{\sqrt{-\varepsilon_1(1-w)^2 + (1-\varepsilon_1 w)^2 \tanh^2\left(k_0 d_1 \sqrt{-\varepsilon_1}\right)}}{\sqrt{w}\left[(1-\varepsilon_1 w)\tanh\left(k_0 d_1 \sqrt{-\varepsilon_1}\right) - i\sqrt{-\varepsilon_1}(w-1)\right]}. \tag{23}$$

Looking at its magnitude:

$$\left|A_3^+\right| = \frac{1}{\sqrt{w}}, \tag{24}$$

and recalling the conditions in equations (19) and (20), together with the assumed unit-amplitude excitation, it would seem at a first glance that power conservation is violated, as $|A_3^+| > 1$ (for $w < 1$) or $|A_3^+| < 1$ (for $w > 1$). This seeming paradox is resolved by taking into account the infinite extent of the slab and of the excitation, and the character of the mapping function in equation (21), which resembles the finite embedded coordinate transformations utilized in Rahm *et al.* (2008) for the synthesis of beam compressors/expanders. This implies the appearance of a net incoming (for $w < 1$) or outgoing (for $w > 1$) power flux guided by the transformation slab along the $y$-direction, which balances the difference between impinging and transmitted power. This phenomenon is better understood in the presence of spatially-truncated collimated beam illumination, for which we anticipate interesting funneling (for $w < 1$) or defunneling (for $w > 1$) effects.

## 4. Representative numerical results

Consider, for example, an ENG slab with $\varepsilon_1 = -2$ and $d_1 = 0.1\lambda_0$. A possible parameter configuration for the DPS transformation slab satisfying the zero-reflection condition in equation (13) is given by $w = 0.0517$ (by closely fulfilling the constraint in equation (19)), $U_0 = 2.48$, and $d_2 = 0.246\lambda_0$.

Figure 2 shows the relevant constitutive parameters of the corresponding transformation slab, obtained using equation (1), with equation (2) and equation (21). More specifically, the figure shows the spatial distribution of the diagonalized relative permittivity tensor components (eigenvalues of equation (1)) in the local principal coordinate system (with the local eigenvector directions indicated as short segments), from which the inhomogeneous, anisotropic, DPS character is fairly evident. Note that, as for certain classes of transformation slabs illustrated in Gallina *et al.* (2010), such parameters tend to exhibit extreme values for $|y| \to \infty$.

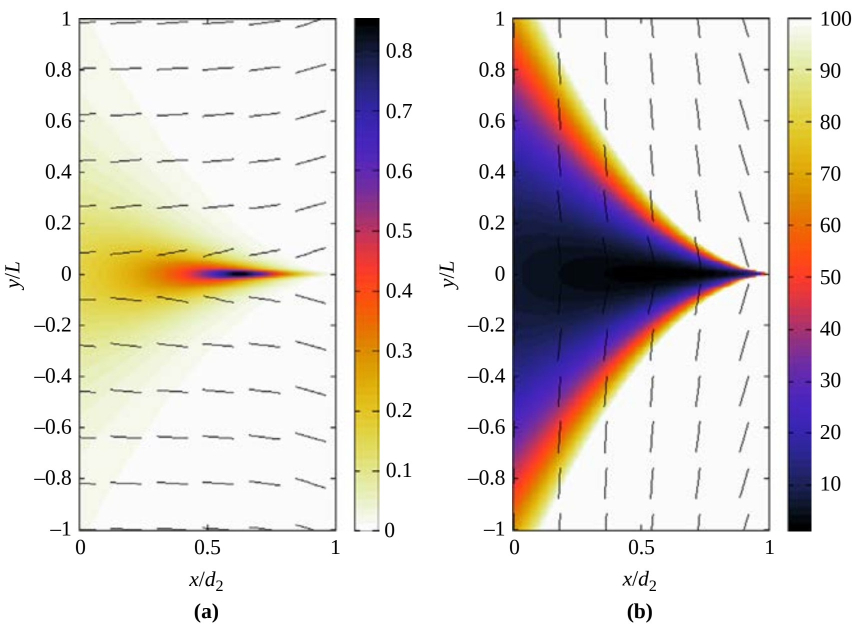
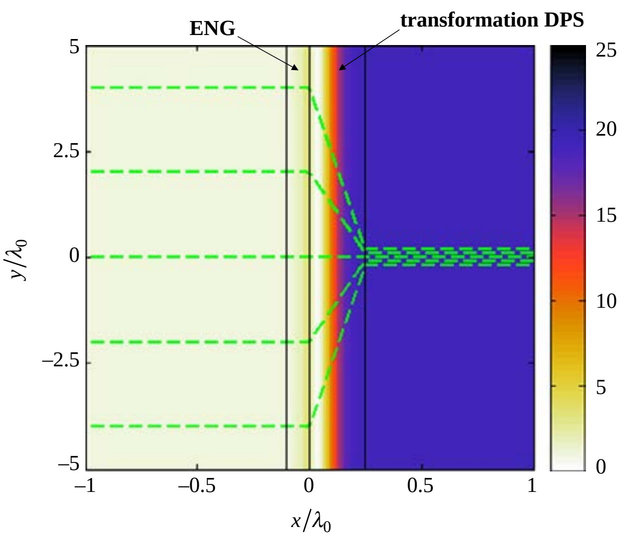



**Figure 2.**
Relevant (in-plane) components of the relative-permittivity tensor pertaining to the transformation slab arising from equation (21) (with $w = 0.0517$, $U_0 = 2.48$, and $d_2 = 0.246\lambda_0$), shown in the principal reference system (with local axes directions shown as short segments)

However, this does not impose a strong limitation, in view of the unavoidable spatial truncation of the slab and the excitation.

With reference to the ideal, lossless scenario above, Figure 3 shows the magnetic field intensity map computed from the analytical model in Section 3, with overlaid some representative streamlines, which illustrate the power flow through and along the bi-layer. In particular, while the magnetic field distribution resembles a standing wave, the streamlines clearly display the anticipated funneling effect, with an incoming power flow guided by the transformation slab that generates a transmitted wave of intensity larger than the impinging one.

As previously mentioned, to better illustrate the funneling phenomenon, it is more insightful to study a spatially-confined beam-type (rather than plane-wave) excitation. In this framework, we illustrate below the results from a numerical study conducted via the finite-element-based COMSOL Multiphysics software package (COMSOL, 2007). Accordingly, in our numerical simulations, the bi-layer slab is truncated along the $y$-direction to a width $2L = 10\lambda_0$, and the relative-permittivity parameters of the transformation slab (whenever unrealistically high) are truncated to a maximum value of 100 (Figure 2). Moreover, a loss-tangent of $10^{-2}$ is assumed for all materials. Figure 4 shows the magnetic-field intensity map pertaining to the ENG-DPS paired configuration above excited by a unit-amplitude, normally-incident collimated Gaussian beam with waist size of $5\lambda_0$, from which a sensible transmission accompanied by a funneling effect is now more evident. More quantitative information can be gathered from the on-axis field distribution shown in Figure 5, which shows the field growth in the bi-layer. As an effect of the material losses and geometrical/parameter truncations, we note a moderately lower (compared to that in Figure 4) transmitted-field intensity, as well as




**Notes:** Also shown are some representative (dashed) streamlines indicating the local energy flux (i.e. Poynting vector)

**Figure 3.**
(a) Magnetic field intensity map (in arbitrary units) for an ENG slab with $\varepsilon_1 = -2$ and $d_1 = 0.1\lambda_0$ paired with a transformation slab with parameters as in Figure 2, under normally-incident plane-wave illumination

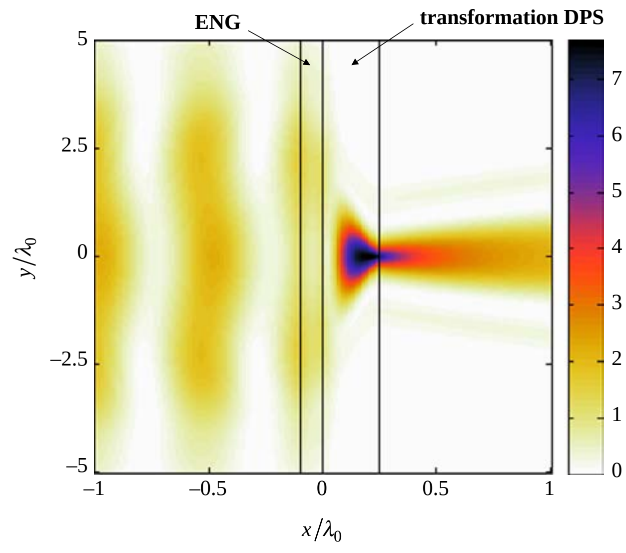



**Figure 4.**
As in Figure 3, but under a collimated Gaussian-beam (with waist size of $5\lambda_0$) illumination, assuming truncation of the transformation-slab constitutive parameters in Figure 2 to a maximum value of 100, and a loss-tangent of 0.01 for all materials

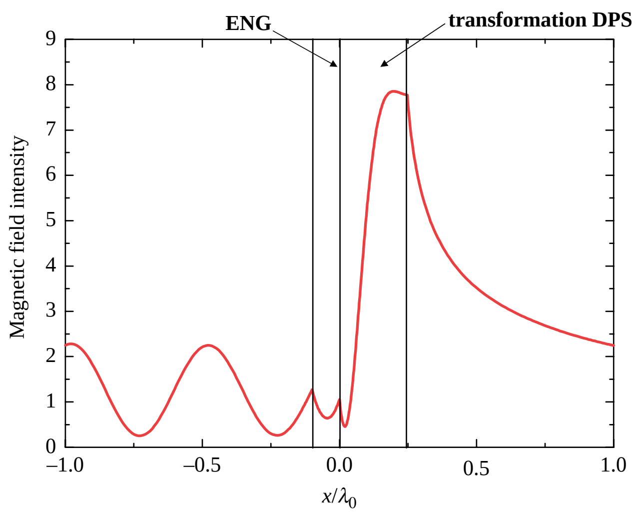



**Figure 5.**
On-axis ($y = 0$) magnetic field intensity distribution from Figure 4

the standing-wave pattern in the $x < 0$ vacuum halfspace, due to reflections. Complementary de-funneling effects, not shown here for brevity, may be observed for the $w > 1$ case.

## 5. Conclusions

In summary, we have shown here an interesting, counterintuitive phenomenon that can take place when pairing a homogenous, isotropic ENG slab with an inhomogeneous, anisotropic DPS slab. In spite of the strongly reflective character that the single constituents would generally exhibit when taken alone, it is possible via a suitable transformation-optics-inspired design to achieve zero-reflection from a (lossless) ENG-DPS bi-layer, under normally-incident plane-wave illumination, at a given frequency. For spatially-truncated collimated beam excitation, and in the presence of moderate material losses and parameter truncations, one may still obtain a substantial transmission of the impinging power accompanied by a spatial reshaping of its spatial distribution, which yields a funneling or defunneling effect.

Building up on our previous studies (Castaldi *et al.*, 2010, 2011a), the results presented here provide further evidence and insight in how spatial inhomogeneity and anisotropy in DPS media may be engineered to emulate (within narrow parametric ranges) SNG-like responses. Moreover, they also show that the powerful framework of transformation optics may be fruitfully exploited to engineer resonant transmission phenomena through SNG materials. Current and future investigations are aimed at exploring feasible implementations (e.g. via layered (Chen and Chan, 2008) or wire media (Alekseyev *et al.*, 2010)) of the arising transformation medium, so as to account for realistic dispersion effects.

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